Solutions to Quiz 3

1. Consider the following two-point boundary value problem

$$y'' + 16y = 0, \qquad y(0) = 7, \qquad y(A) = B.$$

For which values of A and B, does the boundary value problem have a unique solution?

A. $A = \frac{\pi}{4}$ and $B = 7$

B. $A = \frac{\pi}{4}$ and $B = -7$

C. None of these

D. $A = \frac{\pi}{8}$ **and** $B = 5$

> **Solution:** The general solution to $y'' + 16y = 0$ is $y(x) = C_1 \cos(4x) + C_2 \sin(x)$. We have $y(0) = C_1 = 7$. If $A = \frac{\pi}{4}$, then $y(\frac{\pi}{4}) = -7 = B$. In this case, if $B = -7$ then there are infinitely many solutions. If $B \neq -7$, then there is no solution. If $A = \frac{\pi}{8}$, then $y(\frac{\pi}{8}) = C_2 = B$. Thus, no matter what B is, there exists a solution and it is unique.

2. If $f(x) = -4(x^2 - C)$ and $g(x) = 7$ are orthogonal on $[-1, 1]$, then what is C?

A. $\frac{1}{3}$

B. 1

C. 0

D. 2

E. $\frac{1}{2}$

> **Solution:** Since f and g are orthogonal, we have
>
> $$\begin{aligned}\int_{-1}^{1} f(x)g(x)\,dx &= -28\int_{-1}^{1}(x^2 - C)\,dx \\ &= -28[\frac{1}{3}x^3 - Cx]_{-1}^{1} \\ &= -28[\frac{2}{3} - 2C] \\ &= 0.\end{aligned}$$
>
> Thus, $C = \frac{1}{3}$.

3. If f is periodic with period 12 and $f(x) = -8x^2 + 8$ for $x \in (-6, 6]$, then

$$f(x) = px^2 + qx + r$$

for $x \in (6, 18]$. Find p, q, r.

Solution: Since f is periodic, we have $f(x) = f(x+12) = f(x-12) = \cdots$. Let $x \in (6, 18]$ then $x - 12 \in (-6, 6]$. Thus,

$$f(x) = f(x-12) = -8(x-12)^2 + 8 = -8x^2 + 192x - 1144.$$

4. What is the fundamental period of the function

$$f(x) = 4\cos\left(\frac{8\pi}{4}\,x\right) + 3\sin\left(\frac{96\pi}{4}\,x\right)$$

A. $T = \frac{8}{12}$
B. $T = 2\pi$
C. $T = \frac{8}{8}$
D. $T = \pi$
E. $T = 8$
F. $T = 2\pi$
G. $T = \frac{12}{8}$
H. None of these

Solution: The fundamental period is the smallest of the periods of $\cos\left(\frac{8\pi}{4}\,x\right)$and $\sin\left(\frac{96\pi}{4}\,x\right)$, since they are one multiple of the other and since the period of $\cos(\frac{n\pi}{L}x)$ and $\cos(\frac{n\pi}{L}x)$ is $T = \frac{2L}{n}$. Thus he period of $\cos\left(\frac{8\pi}{4}\,x\right)$and $\sin\left(\frac{96\pi}{4}\,x\right)$ are respectively $T = \frac{8}{8} = 1$ and $T = \frac{8}{96} = 18$. Thus the period is $T = \frac{8}{8} = 1$.

5. What the solution to the boundary value problem

$$y'' + \lambda y = 0;\ y(0) = 0;\ y(4) = 0$$

A. $y_n = \sin\left(\frac{n\pi x}{4}\right)$
B. $y_n = \cos\left(\frac{n\pi x}{4}\right)$
C. $y_n = \sin\left(\frac{n\pi x}{8}\right)$
D. $y_n = c_1\sin\left(\frac{n\pi x}{4}\right) + c_2\cos\left(\frac{n\pi x}{4}\right)$
E. None of these

Solution: The solution to $y'' + \lambda y = 0;\ y(0) = 0;\ y(L) = 0$ is $y_n = \sin\left(\frac{n\pi x}{L}\right)$

6. Does the existence and uniqueness theorem apply to boundary value problems of the form

$$y'' + ay' + by = 0,\ y(2) = y(4) = 0$$

with a and b real numbers?

A. No choice Yes

Solution: The existence and uniqueness theorem for ordinary differential equation applies to initial conditions which are assigned at the same point. Here they are assigned at two distinct points