# Solutions to Quiz 2

1. Suppose

$$t^2y'' - 15ty' + 64y = 0$$

has a solution $y_1(t) = t^8$. If $y_2(t) = t^8v(t)$ is another solution, then $v(t)$ satisfies

A. $t^2v'' = -tv' + 8v$

B. $tv'' = v'$

C. $v'' = -tv'$

D. None of these

**E.** $tv'' = -v'$

**Solution:** Direct computation yields

$$\begin{aligned} t^2y_2'' - 15ty_2' + 64y_2 &= t^2(56t^6v(t) + 16t^7v'(t) + t^8v''(t)) - 15t(8t^7v(t) + t^8v'(t)) + 64t^8v(t) \\ &= 16t^9v'(t) + t^{10}v''(t) - 15t^9v'(t) \\ &= t^9(v'(t) + tv''(t)) \\ &= 0. \end{aligned}$$

2. Identify the correct form of a particular solution to

$$y'' - 6y' = te^{-6t} + 4t^4.$$

A. $A_0 + A_1t + A_2t^2 + A_3t^3 + A_4t^4 + B_0e^{-6t} + B_1te^{-6t}$

B. $A_0 + A_1t + A_2t^2 + A_3t^3 + A_4t^4 + B_0te^{-6t} + B_1t^2e^{-6t}$

**C.** $A_0t + A_1t^2 + A_2t^3 + A_3t^4 + A_4t^5 + B_0e^{-6t} + B_1te^{-6t}$

D. None of these.

E. $A_0t + A_1t^2 + A_2t^3 + A_3t^4 + A_4t^5 + B_0te^{-6t} + B_1t^2e^{-6t}$

**Solution:** Let $L[y] = y'' - 6y'$. First, the fundamental solutions for $L[y] = 0$ are $y_1 = 1$ and $y_2 = e^{6t}$. A particular solution to $L[y] = te^{-6t}$ has the form

$$(B_0 + B_1t)e^{-6t}.$$

For $L[y] = 4t^4$, $A_0 + A_1t + A_2t^2 + A_3t^3 + A_4t^4$ won't work because $A_0$ is a solution to $L[y] = 0$. Thus,

$$t(A_0 + A_1t + A_2t^2 + A_3t^3 + A_4t^4)$$

is a particular solution to $L[y] = 4t^4$. Therefore, $A_0t + A_1t^2 + A_2t^3 + A_3t^4 + A_4t^5 + B_0e^{-6t} + B_1te^{-6t}$ is the correct form of a particular solution.

3. If $u(t) = 3f(t) + 6g(t)$ and $v(t) = -5f(t) + g(t)$, then the Wronskian

$$W[u, v](t) = kW[f, g](t)$$

for all real numbers $t$. What is $k$?

**Solution:** We directly see

$$\begin{aligned} W[u, v](t) &= (3f(t) + 6g(t))(-5f(t) + g(t))' - (3f(t) + 6g(t))'(-5f(t) + g(t)) \\ &= -15ff' + 3fg' - 30f'g + 6gg' + 15ff' - 3f'g + 30fg' - 6gg' \\ &= 33(fg' - f'g) \\ &= 33W[f, g](t). \end{aligned}$$

4. Can $y = \cos(-t) + \sin(-2t)$ be a solution to a second order linear homogeneous differential equation

$$y'' + ay' + by = 0$$

with $a$ and $b$ real numbers?

**Solution:** No, it cannot. A function $y = c_1 \cos(\alpha_1 t) + c_2 \sin(\alpha_2 t)$ with $\alpha_1 \neq \alpha 2$ cannot be a solution to an equation $y'' + ay' + by = 0$. Such an equation can only have a solution containing cosine and sine if $r^2 + ar + b = 0$ has purely imaginary roots (if there is a real part to the root, it give rise to an exponential piece) $r = \pm i\alpha$ which would entail solutions of the form $y = c_1 \cos(\alpha t) + c_2 \sin(\alpha t)$, with the arguments of sin and cos equal to one another

5. Consider the following ODE for $y(x)$:

$$y'' - 6y' + 234y = 0.$$

Which of these is a correct form for the general solution?

**A.** $y = e^{3x} (A \cos(15x) + B \sin(15x))$
B. $y = Ae^{3x} + Be^{43x}$
C. $y = e^{15x} (A \cos(3x) + B \sin(3x))$
D. $y = A \cos(3x) + B \sin(3x)$
E. $y = Ae^{3x} + B \cos(15x) + C \sin(15x)$
F. $y = Axe^{3x} + Be^{43x}$
G. $y = Ax + B + Ce^{3x} + De^{15x}$
H. None of these

**Solution:** The characteristic polynomial $r^2 - 6r + 234$ has roots $r = 3 \pm 15i$ and therefore the general solution is $y = e^{3x}\,(A\cos(15x) + B\sin(15x))$.

6. Consider the following ODE for $y(x)$:

$$y'' + 6y' + 9y = 0.$$

Which of these is a correct form for the general solution?

**A.** $y = Ae^{-3x} + Bx\,e^{-3x}$

B. $y = Ae^{-3x} + Be^{44x}$

C. $y = Ae^{-3x} + Be^{14x} + Cx$

D. $y = Ax + B$

E. $y = Ae^{-3x} + Be^{44x} + Ce^{14x}$

F. $y = Axe^{-3x} + Bxe^{44x}$

G. $y = Ax + B + Ce^{-3x} + De^{14x}$

H. None of these

**Solution:** The characteristic polynomial $r^2 + 6r + 9 = (r+3)^2$ has the repeated root $r = -3$ and therefore the general solution is $y = Ae^{3x} + Bx\,e^{-3x}$.