Math 285 Midterm 2 Practice Exam

1. The existence and uniqueness theorem guarantees that there exists a unique solution of the initial value problem $(t-1)(t+3)y''' + ty' + te^{-t^2}y = \ln|t-4|$ with $y(2) = 2$, $y'(2) = 1$, and $y''(2) = 0$ on an open interval

   A. $(1, 4)$

   B. $(-\infty, -3)$

   C. $(-3, 1)$

   D. $(4, \infty)$

   E. None of these

2. If the general solution of $y'' + py' + qy = 0$ is $y(t) = e^{3t}(C_1 \cos 2t + C_2 \sin 2t)$, what are the value of p and q?

   A. $p = -6$ and $q = 5$

   B. $p = -6$ and $q = 13$

   C. $p = 6$ and $q = 13$

   D. $p = 6$ and $q = 5$

   E. None of these

3. The initial value problem

$$u'' + 4u = 3\cos t$$

   with $u(0) = u'(0) = 0$ has the solution $u(t) = \cos t - \cos 2t$. This can be written as a product of trigonometric functions

   A. $2\cos\left(\frac{3}{2}t\right)\sin\left(\frac{1}{2}t\right)$

   B. $\sin\left(\frac{3}{2}t\right)\sin\left(\frac{1}{2}t\right)$

   C. $2\sin\left(\frac{3}{2}t\right)\sin\left(\frac{1}{2}t\right)$

   D. $\sin\left(\frac{3}{2}t\right)\cos\left(\frac{1}{2}t\right)$

   E. None of these

4. Identify the correct form of a particular solution for the following differential equation.

$$y'' + 2y' = te^{2t} + 10t^3.$$

   A. $Y(t) = At^2e^{2t} + Bte^{2t} + C + Dt + Et^2 + Ft^3$

   B. $Y(t) = Ate^{2t} + Be^{2t} + C + Dt + Et^2 + Ft^3$

   C. $Y(t) = At^2e^{2t} + Bte^{2t} + Ct + Dt^2 + Et^3 + Ft^4$

   D. $Y(t) = Ate^{2t} + Be^{2t} + Ct + Dt^2 + Et^3 + Ft^4$

E. None of these

5. For which value of k is the following oscillator in resonance?

$$u'' + ku = 3\cos(2t).$$

A. $k = 1$
B. $k = 2$
C. $k = 9$
D. None of these
E. $k = 4$

6. The differential equation

$$u'' + 5u' + 9u = 0$$

corresponds to an oscillator that is

A. undamped
B. underdamped
C. overdamped
D. critically damped
E. None of these

7. The function $u(t) = -3\sqrt{3}\cos t + 3\sin t$ can be written as $u(t) = R\cos(\omega_0 t - \delta)$ where

A. $R = 6$, $\omega_0 = 1$, $\delta = 11\pi/6$
B. $R = 3$, $\omega_0 = 1$, $\delta = \pi/6$
C. $R = 3$, $\omega_0 = 1$, $\delta = 7\pi/6$
D. $R = 6$, $\omega_0 = 1$, $\delta = 5\pi/6$
E. None of these

8. Which of these is NOT a set of linearly independent solutions?

A. $e^x, 2e^x - e^{2x}, e^x + 3e^{2x}$
B. $x, x^2 + x, 2x^3$
C. $\cos x, \sin x, \cos 2x$
D. $2, x, x\ln x$
E. None of these

9. The third order differential equation

$$y''' + py'' + qy' + ry = 0$$

has the characteristic equation $(\lambda + 1)(\lambda^2 + 4\lambda + 5) = 0$. What is the general solution to the differential equation?

A. $C_1e^{-t} + C_2e^{t} + C_3e^{-5t}$

B. $C_1e^{t} + C_2e^{-2t}\cos t + C_3e^{-2t}\sin t$

C. $C_1e^{-t} + C_2e^{-2t}\cos t + C_3e^{-2t}\sin t$

D. $C_1e^{t} + C_2e^{t} + C_3e^{-5t}$

E. None of these

10. Find the solution to the following initial value problem

$$y'' - 6y' + 9y = 2te^{3t}, \qquad y(0) = 1, \qquad y'(0) = 0.$$

11. Consider $t^2y'' - 5ty' + 9y = 0$ for $t > 0$.

    1. Find r such that $y_1(t) = t^r$ is a solution to the equation.
    2. Find another solution y_2 to the equation such that $W[y_1, y_2](t) \neq 0$. (Hint: use the method of reduction of order.)