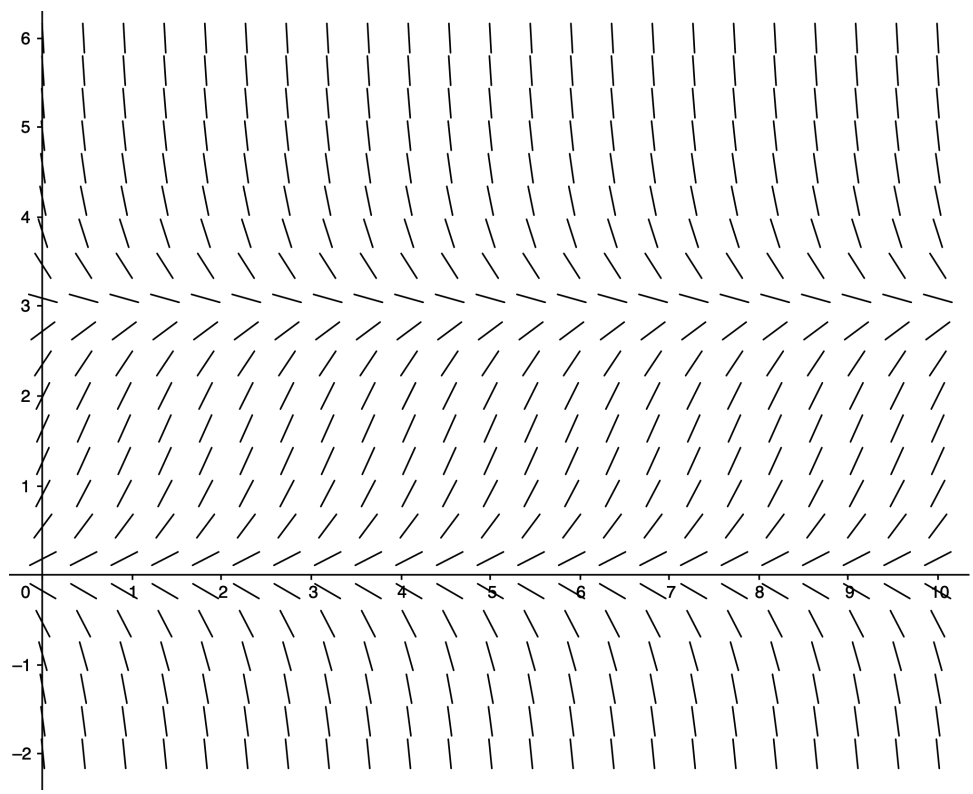
Math 285 Midterm 1 Practice Exam

1. Identify which differential equation below corresponds to the following direction field.



A. $y' = y(3 - y)$
B. $y' = y(y - 3)$
C. $y' = y(y - 2)$
D. $y' = y - 3$
E. None of these

2. Find the solution of

$$\frac{dy}{dt} = 5 - 2y.$$

A. $y(t) = Ce^{-2t} + \frac{5}{2}$ where C is an arbitrary constant.
B. $y(t) = Ce^{-t} + \frac{5}{2}$ where C is an arbitrary constant.
C. $y(t) = e^{-2t} + C$ where C is an arbitrary constant.

D. $y(t) = e^{-t} + C$ where C is an arbitrary constant.

E. None of these

3. How would you classify the following equation for $y(t)$?

$$(1+t^2)(y''')^2 + e^t \ln y = \frac{y}{t}.$$

A. A third order nonlinear differential equation.

B. A third order linear differential equation.

C. A sixth order nonlinear differential equation.

D. A sixth order linear differential equation.

E. None of these

4. What is the correct integrating factor to solve the following ODE for $y(t)$?

$$t^4 y' + 3t^3 y + 1 = 0.$$

A. $\mu(t) = t^3$

B. $\mu(t) = \frac{1}{t^3}$

C. $\mu(t) = 3t$

D. $\mu(t) = \frac{3}{t}$

E. None of these

5. Solve the following initial value problem.

$$y' + 3y = e^{-t}, \qquad y(1) = 0.$$

A. $y(t) = \dfrac{1}{2}(e^{-t} - e^{2-3t})$

B. $y(t) = -e^{-4t} + e^{-3t-1}$

C. $y(t) = \dfrac{1}{2}(e^{2t} - e^{2})$

D. $y(t) = e^{-t} - e^{-1}$

E. None of these

6. The following nonlinear equation for $y(x)$ can be transformed with a substitution into which separable equation for $v(x)$?

$$y' = \frac{-x^2 + 2xy + y^2}{x^2} + \sec\left(\frac{y}{x}\right)$$

A. $xv' = -1 + v + v^2 + \sec v$

B. $v' = -1 + 2v + v^2 + \sec v$

C. $xv' = -x^2 + v + v^2 + \sec v$

D. $v' = 1 - 2v + v^2$

E. None of these

7. Determine the values of r for which the given differential equation has solutions of the form $y(t) = e^{rt}$.

$$y'' + 4y' - 5y = 0.$$

A. $r = 1, -5$

B. $r = -1, 5$

C. $r = 1, 5$

D. $r = 3, -5$

E. None of these

8. The existence and uniqueness theorem for linear differential equations ensures that the solution of

$$(t-2)(t+3)y' + \sqrt{t}y = 1 + t^2, \qquad y(1) = -1$$

exists for all t in the region defined by:

A. $(0, 2)$

B. $(-3, 2)$

C. $(2, \infty)$

D. $(-\infty, -3)$

E. None of these

9. Consider the following nonlinear ODE for $y(x)$. Which initial condition would guarantee a unique solution to the initial value problem?

$$yy' + x^2 y' = \frac{e^{-x}}{x+1}$$

A. $y(2) = 4$

B. $y(2) = -4$

C. $y(-1) = 1$

D. $y(-1) = -1$

E. None of these

10. For which values of r and K, is the constant function $y(t) = -2$ an asymptotic stable equilibrium solution to the following equation?

$$y' = r(y+2)(y-K).$$

A. $r = 1$ and $K = 0$

B. $r = 2$ and $K = -3$

C. $r = -1$ and $K = -2$

D. $r = -2$ and $K = 1$

E. None of these

11. Consider the following differential equation.

$$y' = \frac{x^4 + 3y}{x}, \qquad x > 0.$$

1. Determine whether the equation is linear. Give a reason why.
2. Solve the equation with the initial condition $y(2) = 4$.

12. Find the solution to the following initial value problem.

$$(x^4 + 1)y' = 2x^3y^2, \qquad y(0) = \frac{3}{2}$$

and specify the interval where the solution is defined.