Final Exam Practice Problems

July 23, 2024

Midterm 1

A box in a certain supply room contains four 40W light bulbs, five 60W bulbs, and six 75W bulbs.

1. Suppose that three bulbs are randomly selected. What is the probability that exactly two of the selected bulbs are rated 75W?
2. Suppose now that bulbs are to be selected one by one until a 75W bulb is found. What is the probability that it is necessary to examine at least six bulbs?

Midterm 1

One percent of all individuals in a certain population are carriers of a particular disease. A diagnostic test for this disease has a 90% detection rate for carriers and a 5% detection rate for noncarriers. Suppose the test is applied independently to two different blood samples from the same randomly selected individual.

1. What is the probability that both tests are positive?
2. Given that both tests are positive, what is the conditional probability that the selected individual is a carrier?

Midterm 1

A toll bridge charges \$1.00 for passenger cars and \$2.50 for other vehicles. Suppose that during daytime hours, 60% of all vehicles are passenger cars. Suppose 25 vehicles cross the bridge during a particular daytime period.

1. Let X be the number of passenger cars among 25 vehicles. How is X distributed? Justify your answer.
2. What is the expected number of passenger cars among 25 vehicles, that is, $\mathbb{E}[X]$?
3. Find the expression of the toll revenue from the 25 vehicles $h(X)$ in terms of X. Compute the expected revenue $\mathbb{E}[h(X)]$.

Midterm 1

Two fair six-sided dice are tossed independently. Let X be the maximum of the two outcomes. (For example, if two outcomes are 1 and 5, then $X = 5$.)

1. Let $p(x)$ be the PMF of X. Find $p(3)$ and $p(4)$.
2. Let $F(x)$ be the CDF of X. Find $F(0)$ and $F(3.2)$.

Midterm 2

Let X and Y be discrete random variables with joint PMF $p(x, y)$ given by

$$p(0,0) = p(0,1) = p(1,1) = p(1,2) = \frac{1}{4}.$$

1. Find the conditional expectation $\mathbb{E}[X|Y = 1]$.
2. Find the covariance $\mathrm{Cov}(X, Y)$.
3. Are they independent?

Midterm 2

Let X, Y be continuous random variables with joint PDF

$$f(x,y) = \begin{cases} Cxy, & 0 \le x \le 1, 0 \le y \le 1, y \ge x \\ 0 & \text{otherwise} \end{cases}$$

for some constant $C > 0$.

1. Find the constant C.
2. Find the marginal PDF of Y.

Midterm 2

Let X be a continuous random variable with CDF

$$F(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{2}x^2 & 0 \leq x < 1, \\ \frac{1}{2}x & 1 \leq x < 2, \\ 1 & x \geq 2. \end{cases}$$

1. Find $\mathbb{P}(0.5 \leq X \leq 1.5)$.
2. Find the PDF of X.

Midterm 2

Let $\Phi(x)$ be the CDF of the standard normal distribution.

1. Let $X \sim N(3, 16)$. Write down $\mathbb{P}(1 \leq X < 4)$ in terms of $\Phi(x)$ for $x \geq 0$. Then use the corresponding table to compute the probability.
2. Let X, Y be independent normal random variables with common distribution $N(3, 16)$. Use the corresponding table to compute the probability $\mathbb{P}(1 \leq X < 2, 4 < Y \leq 5)$.

Midterm 2

Let $X \sim \mathrm{Exp}(2)$.

1. For $0 < p < 1$, find the $(100p)$-th percentile of X.
2. Let $h(x)$ be a function defined by

$$h(x) = \begin{cases} 1, & x \leq 1 \\ x, & x > 1. \end{cases}$$

   Compute the expectation $\mathbb{E}[h(X)]$.

In a notched tensile fatigue test on a titanium specimen, the expected number of cycles to first acoustic emission (used to indicate crack initiation) is $\mu = 28,000$, and the standard deviation of the number of cycles is $\sigma = 5000$.

Let $X_1, X_2, \cdots, X_{25}$ be a random sample of size 25, where each X_i is the number of cycles on a different randomly selected specimen.

The time that it takes a randomly selected rat of a certain subspecies to find its way through a maze is a normally distributed RV with $\mu = 1.5$ min and $\sigma = .35$ min.

Suppose five rats are selected.

Let $X_1, \cdots, X_5$ denote their times in the maze.

Assuming the X_i's to be a random sample from this normal distribution,

what is the probability that the total time is between 6 and 8 min?

A certain consumer organization customarily reports the number of major defects for each new automobile that it tests.

Suppose the number of such defects for a certain model is a random variable with mean value 3.2 and standard deviation 2.4.

Among 100 randomly selected cars of this model, how likely is it that the sample average number of major defects exceeds 4?

(5.4-56) A binary communication channel transmits a sequence of "bits" (0s and 1s). Suppose that for any particular bit transmitted, there is a 10% chance of a transmission error (a 0 becoming a 1 or a 1 becoming a 0).

Assume that bit errors occur independently of one another.

1. Consider transmitting 1000 bits. What is the approximate probability that at most 125 transmission errors occur?
2. Suppose the same 1000-bit message is sent two different times independently of one another.
   What is the approximate probability that the number of errors in the first transmission is within 50 of the number of errors in the second?

A certain automobile manufacturer equips a particular model with either a six-cylinder engine or a four-cylinder engine.

Let X_1 and X_2 be fuel efficiencies for independently and randomly selected six-cylinder and four-cylinder cars, respectively, with

$$\mu_1 = 22, \qquad \mu_2 = 26, \qquad \sigma_1 = 1.2, \qquad , \sigma_2 = 1.5.$$

Find $\mathbb{E}[X_1 - X_2]$ and $\mathrm{Var}(X_1 - X_2)$.

(5.5-62) Manufacture of a certain component requires three different machining operations.

Machining time for each operation has a normal distribution, and the three times are independent of one another.

The mean values are 15, 30, and 20 min, respectively, and the standard deviations are 1, 2, and 1.5 min, respectively.

What is the probability that it takes at most 1 hour of machining time to produce a randomly selected component?

Let μ (a parameter) denote the true average breaking strength of wire connections used in bonding semiconductor wafers.

A random sample of $n = 10$ connections might be made, and the breaking strength of each one determined, resulting in observed strengths $x_1, x_2, \cdots, x_{10}$.

The sample mean breaking strength $\bar{x}$ could then be used to draw a conclusion about the value of μ.

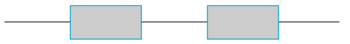
Let $X_1, \cdots, X_n$ be a random sample from a uniform distribution on $[0, \theta]$. Consider

$$\widehat{\theta}_1 = \frac{n+1}{n} \max X_i$$
$$\widehat{\theta}_2 = 2\overline{X}.$$

Are they unbiased?

Find the variances of $\widehat{\theta}_1$ and $\widehat{\theta}_2$ (6.1-8) In a random sample of 80 components of a certain type, 12 are found to be defective.

1. Give a point estimate of the proportion of all such components that are not defective.
2. A system is to be constructed by randomly selecting two of these components and connecting them in series, as shown here.

   The series connection implies that the system will function if and only if neither component is defective (i.e., both components work properly).
   Estimate the proportion of all such systems that work properly.

Let $X_1, \cdots, X_n$ represent a random sample of service times of n customers at a certain facility, where the underlying distribution is assumed exponential with parameter λ.

Find the moment estimator for λ.

Let $X_1, \cdots, X_n$ be a random sample of size n from a Gamma distribution.

Find the moment estimators for α, β.

Let $X_1, \cdots, X_n$ be a random sample from Bernoulli distribution.

Find the Likelihood function and the MLE for p.

Let $X_1, \cdots, X_n$ be a random sample from exponential distribution.

Find the Likelihood function and the MLE for λ.

Let $X_1, \cdots, X_n$ be a random sample from normal distribution.

Find the Likelihood function and the MLEs for μ, σ^2.

(6.2-23) Two different computer systems are monitored for a total of n weeks.

Let X_i denote the number of breakdowns of the first system during the i-th week, and suppose the X_i's are independent and drawn from a Poisson distribution with parameter μ_1.

Similarly, let Y_i denote the number of breakdowns of the second system during the i-th week, and assume independence with each Y_i Poisson with parameter μ_2.

Derive the MLE's of μ_1, μ_2, and $\mu_1 - \mu_2$

Industrial engineers who specialize in ergonomics are concerned with designing workspace and worker-operated devices so as to achieve high productivity and comfort.

A sample of $n = 31$ trained typists was selected, and the preferred keyboard height was determined for each typist.

The resulting sample average preferred height was $\bar{x} = 80.0$ cm.

Assuming that the preferred height is normally distributed with $\sigma = 2.0$ cm, find a Confidence interval for μ.

Extensive monitoring of a computer time-sharing system has suggested that response time to a particular editing command is normally distributed with standard deviation 25 millisec.

A new operating system has been installed, and we wish to estimate the true average response time μ for the new environment.

Assuming that response times are still normally distributed with $\sigma = 25$, what sample size is necessary to ensure that the resulting 95% CI has a width of (at most) 10?

(7.1-4) A CI is desired for the true average stray-load loss μ (watts) for a certain type of induction motor when the line current is held at 10 amps for a speed of 1500 rpm. Assume that strayload loss is normally distributed with $\sigma = 3.0$.

1. Compute a 95% CI for μ when $n = 25$ and $\bar{x} = 58.3$.
2. Compute a 95% CI for μ when $n = 100$ and $\bar{x} = 58.3$.
3. Compute a 99% CI for μ when $n = 100$ and $\bar{x} = 58.3$.
4. How large must n be if the width of the 99% interval for μ is to be 1.0?

The article “Gas Cooking, Kitchen Ventilation, and Exposure to Combustion Products” (Indoor Air, 2006: 65–73) reported that for a sample of 50 kitchens with gas cooking appliances monitored during a one-week period, the sample mean CO2 level (ppm) was 654.16, and the sample standard deviation was 164.43.

Calculate a 95% (two-sided) confidence interval for true average CO2 level in the population of all homes from which the sample was selected.

The charge-to-tap time (min) for carbon steel in one type of open hearth furnace is to be determined for each heat in a sample of size n.

If the investigator believes that almost all times in the distribution are between 320 and 440, what sample size would be appropriate for estimating the true average time to within 5 min. with a confidence level of 95%?