Homework 7

Math 461: Probability Theory, Spring 2022
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Due date: Mar 25, 2022

Instruction

1. Each problem is worth 10 points and only five randomly chosen problems will be graded.
2. Convert a photocopy of your solutions to **one single pdf file** and upload it on Moodle.
3. Please indicate whom you worked with, it will not affect your grade in any way.

1. If X is uniformly distributed over $(0, 1)$, find the density function of $Y = e^X$.
2. If X is uniformly distributed over $(-1, 1)$, find (a) $\mathbb{P}(|X| > 1/2)$ and (b) the density function of the random variable $|X|$.
3. If X is a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$, compute (in terms of the Standard normal CDF $\Phi(\cdot)$)
   (a) $\mathbb{P}(X > 5)$; (b) $\mathbb{P}(4 < X < 16)$; (c) $\mathbb{P}(X < 8)$; (d) $\mathbb{P}(X < 20)$; (e) $\mathbb{P}(X > 16)$.
4. Suppose that the height, in inches, of a 25-year-old man is a normal random variable with parameters $\mu = 71$ and $\sigma^2 = 6.25$. What percentage of 25-year-old men are over 6 feet, 2 inches tall? What percentage of men in the 6-footer club are over 6 feet, 5 inches?
5. The width of a slot of a duralumin forging is (in inches) normally distributed with $\mu = .900$ and $\sigma = .003$. The specification limits were given as $.900 \pm .005$.
   (a) What percentage of forgings will be defective?
   (b) What is the maximum allowable value of σ that will permit no more than 6 in 1000 defectives when the widths are normally distributed with $\mu = .9000$ and σ?
6. One thousand independent rolls of a fair die will be made. Compute an approximation to the probability that the number 6 will appear between 150 and 200 times inclusively. If the number 6 appears exactly 200 times, find the probability that the number 5 will appear strictly less than 150 times.
7. Twelve percent of the population is left handed. Approximate the probability that there are at least 20 left-handers in a school of 200 students. State your assumptions.
8. The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\lambda = \frac{1}{2}$. What is
   (a) the probability that a repair time exceeds 2 hours?
   (b) the conditional probability that a repair takes at least 10 hours, given that its duration exceeds 9 hours?
9. Jones figures that the total number of thousands of miles that an auto can be driven before it would need to be junked is an exponential random variable with mean 20. Smith has a used car that he claims has been driven only 10,000 miles. If Jones purchases the car, what is the probability that she would get at least 20,000 additional miles out of it? Repeat under the assumption that the lifetime mileage of the car is not exponentially distributed, but rather is (in thousands of miles) uniformly distributed over $(0, 40)$.
10. If X is an exponential random variable with parameter $\lambda = 1$, compute the probability density function of the random variable Y defined by $Y = \log X$.