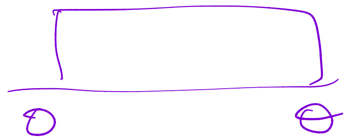


$$= \theta$$

$$E[\hat{\theta}_1] = E\left[\frac{n+1}{n} \max\{X_i\}\right] = \frac{n+1}{n} E[Y] = \frac{n+1}{n} \cdot \frac{n}{n+1}\theta$$

Chap 5.4-7.2

$$E[\bar{X}] = E\left[\frac{1}{n}\sum X_i\right] = E(X_1) = \frac{\theta}{2}$$

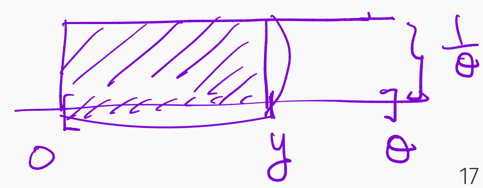
Let $X_1, \cdots, X_n$ be a random sample from a uniform distribution on $[0, \theta]$. Consider

$$\hat{\theta}_1 = \frac{n+1}{n} \max X_i$$
$$\hat{\theta}_2 = 2\bar{X}.$$

Are they unbiased?

Find the variances of $\hat{\theta}_1$ and $\hat{\theta}_2$ ~~(61-8) In a random sample of 80 components of a certain~~

$$E[\underbrace{\max\{X_i\}}_{Y}] = ?$$



CDF

$$F(y) = P(Y \le y) = P(\max\{X_i\} \le y)$$
$$= P(X_1 \le y, \cdots, X_n \le y)$$
$$= (P(X_1 \le y))^n \quad \text{(because i.i.d)}$$
$$= \left(\frac{y}{\theta}\right)^n = \theta^{-n} \cdot y^n$$
$$f(y) = \theta^{-n} \cdot n y^{n-1} \qquad 0 < y < \theta$$

$$E[X_1] = \frac{0+\theta}{2}$$

Chap 5.4-7.2

Let $X_1, \cdots, X_n$ be a random sample from a uniform distribution on $[0, \theta]$. Consider

$$\widehat{\theta}_1 = \frac{n+1}{n} \max X_i$$
$$\widehat{\theta}_2 = 2\overline{X}.$$

Q: Are they unbiased?

Find the variances of $\widehat{\theta}_1$ and $\widehat{\theta}_2$ ~~(6.1-8) In a random sample of 80 components of a certain~~

$$E[\widehat{\theta}_1] =$$

$$E[\widehat{\theta}_2] = E[2\overline{X}] = 2E[\overline{X}] = 2 \cdot \frac{1}{n} E[X_1 + \cdots + X_n]$$

$$= \frac{2}{n}\left(\underbrace{E[X_1]}_{\theta/2} + \cdots + \underbrace{E[X_n]}_{\theta/2}\right) = \cancel{\frac{2}{n}} \cdot \cancel{n} \cdot \frac{\theta}{\cancel{2}}$$

$$= \theta$$

$\widehat{\theta}_2$ is unbiased.

Let $X_1, \cdots, X_n$ be a random sample from a uniform distribution on $[0, \theta]$. Consider

$$\widehat{\theta}_1 = \frac{n+1}{n} \max X_i$$
$$\widehat{\theta}_2 = 2\overline{X}.$$

Are they unbiased?

Find the variances of $\widehat{\theta}_1$ and $\widehat{\theta}_2$ (6.1-8) In a random sample of 80 components of a certain

$$E[\hat{\theta}_1] = E\left[\frac{n+1}{n} \max\{X_1, \cdots, X_n\}\right]$$

$$= \frac{n+1}{n} E[\underbrace{\max\{X_1, \cdots, X_n\}}_{=Y}] = \theta.$$

Conti. RV

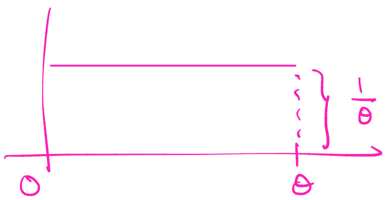
$$E[Y] = \int y \cdot \underbrace{f(y)}_{\text{PDF of } Y} \, dy = \cdots = \boxed{\frac{n}{n+1}\theta}$$

$$\text{CDF of } Y = F(y) = P(Y \le y) = P(\max\{X_1, \cdots, X_n\} \le y)$$

$$= P(X_1 \le y, \cdots, X_n \le y)$$

$$= P(X_1 \le y) \cdots P(X_n \le y) \quad (\text{indep.})$$

$X_1, \cdots, X_n \sim \text{Unif}(0, \theta)$



$$= \left(\frac{y}{\theta}\right) \cdots \left(\frac{y}{\theta}\right) = \left(\frac{y}{\theta}\right)^n \leftarrow \text{CDF}$$

$\Downarrow$

PDF

$$f(y) = \theta^{-n} \cdot n\, y^{n-1}$$

$$P(X_1 \le y) = \int_0^y \frac{1}{\theta}\, dt = \frac{y}{\theta}.$$

$$P(\min\{X_1, \ldots, X_n\} \le y)$$

$$= 1 - P(\min\{X_1, \ldots, X_n\} > y)$$

$$= 1 - P(X_1 > y, X_2 > y, \ldots, X_n > y)$$

$$= 1 - \underbrace{P(X_1 > y)}_{= \left(1 - \frac{y}{\theta}\right)} \underbrace{P(X_2 > y)}_{= \left(1 - \frac{y}{\theta}\right)} \cdots P(X_n > y)$$

$$= 1 - \left(1 - \frac{y}{\theta}\right)^n$$

$$X_1, X_2 \sim \text{Exp}(1)$$

$$Y = \max\{X_1, X_2\}$$

CDF of Y

$$F(y) = \mathbb{P}(Y \le y) = \mathbb{P}(\max\{X_1, X_2\} \le y)$$

$$= \underbrace{\mathbb{P}(X_1 \le y)}_{} \mathbb{P}(X_2 \le y)$$

CDF of X_1

$$= (1 - e^{-y})^2$$