Lecture 1. Basic Combinatorics (Sec 1.1-3)

University of Illinois at Urbana–Champaign

Math 461 Spring 2022

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Why studying probability?

behavior of consumer
depends on

known. →
- financial status
- preference
- age
- ⋮

unknown thing. →
- noise

probability.

complex.

uncertainty
nature.

Why studying probability?

Part 1: What is probability

Part 2: Random variable. (Quantity)

Part 3: Distribution. (Pattern).

Basic Principle of Counting

Question: How many two-letter words are there (using 26 alphabet)?

AA	BA		ZA
AB	BB		ZB
AC	⋮	. . . .	⋮
⋮	⋮		⋮
AZ	BZ		ZZ

$$\underbrace{26 \qquad 26 \qquad \cdots \qquad 26}_{26}$$

$$\boxed{26 \times 26}$$

Basic Principle of Counting

Suppose that two experiments are to be performed.

Then if experiment 1 can result in any one of m possible outcomes

and if, for each outcome of experiment 1, there are n possible outcomes of experiment 2,

then together there are mn possible outcomes of the two experiments.

Basic Principle of Counting

Example

How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers? How many license plates would be possible if repetition among letters or numbers were prohibited?

Question 1

$$26 \times 26 \times 26 \times 10 \times 10 \times 10 \times 10$$

A . . .

AA . . .

AB . . .

B . . .

26

Basic Principle of Counting

Example

How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers? How many license plates would be possible if repetition among letters or numbers were prohibited?

Q2: W/O Repetition.

$$26 \times 25 \times 24 \times 10 \times 9 \times 8 \times 7$$

2^n

Basic Principle of Counting

Example

Consider a set S of n elements, say $S = \{1, 2, \cdots, n\}$. How many different subsets of S are there?

$n=1$: ϕ, $\{1\}$ $\to 2$

not containing 2 / containing 2

$n=2$: ϕ, $\{1\}$, $\{2\}$, $\{1,2\}$ $\to 4$

1 element

$n=3$: ϕ

$\{1\}$, $\{2\}$, $\{3\}$

not

2

$\{1,2\}$, $\{2,3\}$, $\{1,3\}$ } 8

$\{1,2,3\}$ 3

$n=k$ $\quad a_n \to a_{n+1} = 2a_n$, $\quad a_1 = 2$

Permutation

Each ordered arrangement of n distinct objects is called a permutation.

The number of all possible permuations is
$n! = n \cdot (n-1) \cdots 2 \cdot 1.$

factorial.

A B C D
B C D A
C D A B
D A B C
⋮

?

$4 \times 3 \times 2 \times 1$

Permutation

ANS : $4! \times (4! \times 3! \times 2! \times 1!)$

Example

Ms. Jones has 10 books that she is going to put on her bookshelf. Of these, 4 are mathematics books, 3 are chemistry books, 2 are history books, and 1 is a language book. Ms. Jones wants to arrange her books so that all the books dealing with the same subject are together on the shelf. How many different arrangements are possible?

If no constraint : $10!$

By topics.

① Arrange topics: M, C, H, L $\rightarrow 4!$

② $T1$ $4!$ $T2$ $3!$ T_3 $2!$ $T4$ $1!$

$\boxed{M_1 M_2 M_3 M_4}$ $\boxed{C_1 C_2 C_3}$ $\boxed{H_1 H_2}$ $\boxed{L}$

MATH 461 E13/E14 SP22: Probability Theory (Kim, D)

Announcements

On Jan. 19 and Jan. 21, the classes will be in Zoom.

Time: 1-1:50pm

Link: https://illinois.zoom.us/j/85262849161?pwd=UG1ZKzlOTG0wU01COTRGZHpvSVFYUT09

Meeting ID: 852 6284 9161

Password: 594240

Homeworks

Homework 1 (Due: Jan 28, 2021)

MATH 461 E13/E14 SP22: Probability Theory (Kim, D)

Homework 1 (Due: Jan 28, 2021)

① hand-written or Latex ok

Instruction 10

- Each problem is worth 10 points and only five randomly chosen problems will be graded.
- Due is the beginning of the lecture on Jan 28.
- Convert a photocopy of your solutions to one single pdf file. Here are some online free pdf merging/converting services:
  - https://www.pdf2go.com/merge-pdf
  - https://www.pdf2go.com/convert-to-pdf
  - https://www.freepdfconvert.com
- Please indicate whom you worked with, it will not affect your grade in any way.

② readable

③ 1 single pdf

hw1.pdf January 21 2022, 12:53 AM

Submission status

Submission status	No attempt
Grading status	Not graded
Due date	Friday, January 28, 2022, ~~12~~ 1 pm :00 PM
Time remaining	7 days 11 hours
Last modified	-

Add submission

Permutation

Example

How many different letter arrangements can be formed from the letters *arrange*?

ANS : $\frac{7!}{4} = \binom{7}{2,2,1,1,1}$

$4 = 2!\,2!$

$a_1 \quad a_2$

$r_1 \quad r_2$

$n \;,\; g \;,\; e$

$\Rightarrow$ all possible permutation $= 7!$

$\underbrace{a_1 \, a_2}_{2!} \quad \underbrace{r_1 \; n \; r_2}_{2!} \; g \; e$

$a_1 \, a_2 \quad r_2 \; n \; r_1 \; g \; e$

$a_2 \, a_1 \quad r_1 \; n \; r_2 \; g \; e$

$a_2 \, a_1 \quad r_2 \; n \; r_1 \; g \; e$