

# Lecture 1. Basic Combinatorics (Sec 1.1-3)

University of Illinois at Urbana–Champaign

Math 461 Spring 2022

Instructor: Daesung Kim

# Why studying probability?

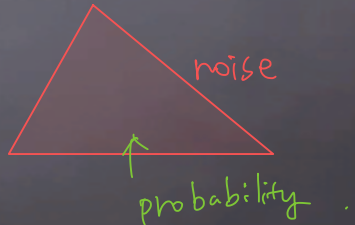
behavior of consumer  
depends on

known. →

financial status  
preference  
age  
⋮

Complex.  
Uncertainty  
nature.

unknown  
thing. →



## Why studying probability?

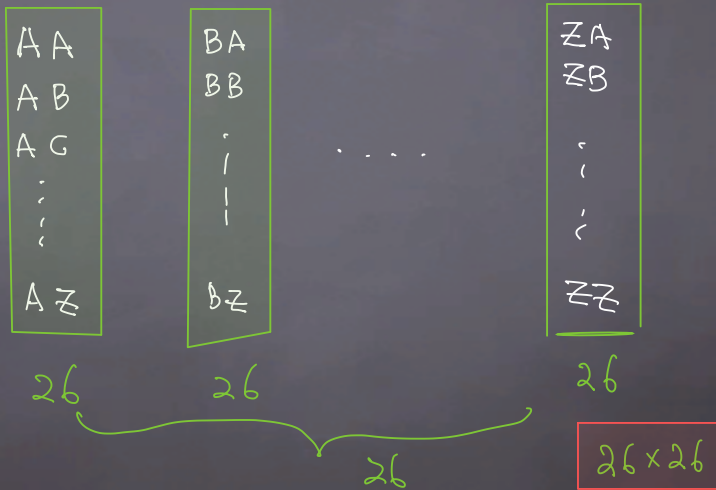
Part 1: What is probability

Part 2: Random variable. (Quantity)

Part 3: Distribution. (Pattern)

## Basic Principle of Counting

**Question:** How many two-letter words are there (using 26 alphabet)?



## Basic Principle of Counting

Suppose that two experiments are to be performed.

Then if experiment 1 can result in any one of  $m$  possible outcomes

and if, for each outcome of experiment 1, there are  $n$  possible outcomes of experiment 2,

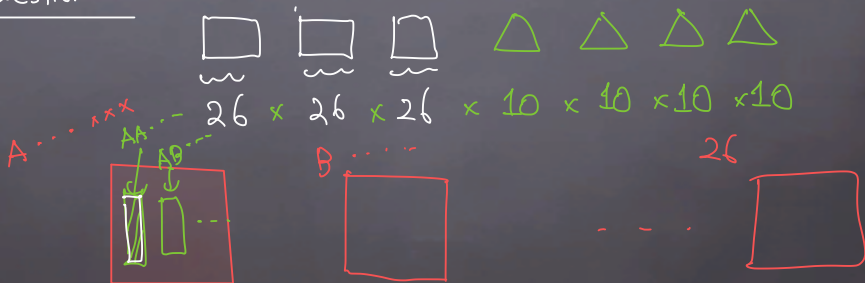
then together there are  $mn$  possible outcomes of the two experiments.

# Basic Principle of Counting

## Example

How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers? How many license plates would be possible if repetition among letters or numbers were prohibited?

### Question 1



## Basic Principle of Counting

### Example

How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers? How many license plates would be possible if repetition among letters or numbers were prohibited?

Q2: w/o Repetition.

$$\begin{array}{ccccccc} \square & \square & \square & \triangle & \triangle & \triangle & \triangle \\ 26 & \times & 25 & \times & 24 & \times & 10 & \times & 9 & \times & 8 & \times & 7 \end{array}$$

$$2^n$$

# Basic Principle of Counting

## Example

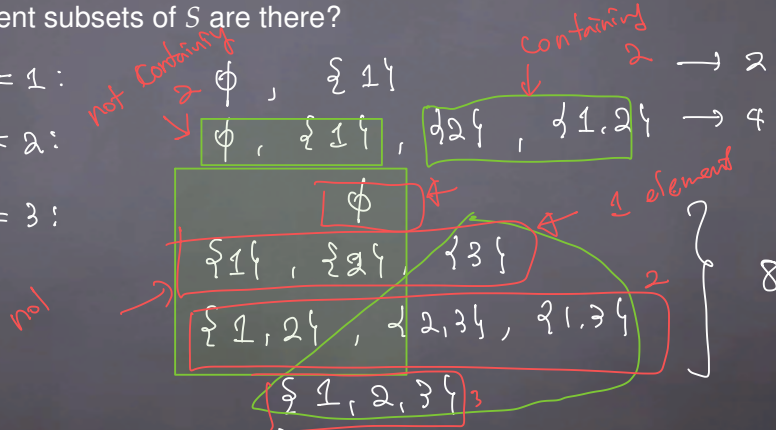
Consider a set  $S$  of  $n$  elements, say  $S = \{1, 2, \dots, n\}$ . How many different subsets of  $S$  are there?

$n = 1$ :  $\emptyset, \{1\}$  *not containing*  $2 \rightarrow 2$  *containing*  $2 \rightarrow 2$

$n = 2$ :  $\emptyset, \{1\}, \{2\}, \{1, 2\}$   $\rightarrow 4$

$n = 3$ :  $\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}$   $\rightarrow 8$

$n = k$   $a_n \rightarrow a_{n+1} = 2a_n, a_1 = 2$





# Permutation

Each ordered arrangement of  $n$  distinct objects is called a **permutation**.

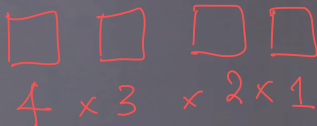
The number of all possible permutations is

$$n! = n \cdot (n-1) \cdot \dots \cdot 2 \cdot 1.$$

factorial.

A	B	C	D
B	C	D	A
C	D	A	B
D	A	B	C
		⋮	
		⋮	
		⋮	

} ?



# Permutation

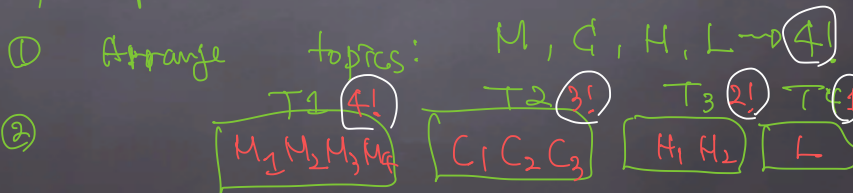
$$\text{ANS : } \underline{4! \times (4! \times 3! \times 2! \times 1!)}$$

## Example

Ms. Jones has 10 books that she is going to put on her bookshelf. Of these, 4 are mathematics books, 3 are chemistry books, 2 are history books, and 1 is a language book. Ms. Jones wants to arrange her books so that all the books dealing with the same subject are together on the shelf. How many different arrangements are possible?

If no constraint :  $10!$

By topics.



# MATH 461 E13/E14 SP22: Probability Theory (Kim, D)

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## Announcements

On Jan. 19 and Jan. 21, the classes will be in Zoom.

Time: 1-1:50pm

Link: <https://illinois.zoom.us/j/85262849161?pwd=UG1ZKzIOTG0wU01COTRGZHpvSVFYUT09>

Meeting ID: 852 6284 9161

Password: 594240

[Announcements](#)


[Syllabus](#)

[Exams](#)

**[Homeworks](#)**

[Quizzes](#)

## Homeworks

 [Homework 1 \(Due: Jan 28, 2021\)](#)

# MATH 461 E13/E14 SP22: Probability Theory (Kim, D)

Dashboard / My courses / MATH 461 E13/E14 SP22 / Homeworks / Homework 1 (Due: Jan 28, 2021)

## Homework 1 (Due: Jan 28, 2021)

### Instruction

- Each problem is worth 10 points and only five randomly chosen problems will be graded.
- Due is the beginning of the lecture on Jan 28.
- Convert a photocopy of your solutions to one single pdf file. Here are some online free pdf merging/converting services:
  - <https://www.pdf2go.com/merge-pdf>
  - <https://www.pdf2go.com/convert-to-pdf>
  - <https://www.freepdfconvert.com>
- Please indicate whom you worked with, it will not affect your grade in any way.

hw1.pdf

January 21 2022, 12:53 AM

### Submission status

Submission status No attempt

Grading status Not graded

Due date Friday, January 28, 2022, 12:00 PM

Time remaining 7 days 11 hours

Last modified -

Add submission

- ① hand-written or latex ok
- ② readable
- ③ 1 single pdf

1 pm

# Permutation

## Example

How many different letter arrangements can be formed from the letters arrange?

ANS :

$$\frac{7!}{(2, 2, 1, 1, 1)} = \frac{7!}{(4) = 2! 2!}$$

$a_1 a_2$

$r_1 r_2$

$n, g, e$

all possible  
permutation



$$= 7!$$

$a_1 a_2$	$r_1 n r_2$	$g e$
$a_1 \bar{a}_2$	$r_2 n r_1$	$g e$
$a_2 a_1$	$r_1 n r_2$	$g e$
$a_2 \bar{a}_1$	$r_2 n r_1$	$g e$

*(Note: In the original image, the first two columns of the table are circled in red, and a bracket groups the first two rows. The circled items are labeled with 2! above them.)*

