Random Box

Consider two boxes, one containing 10 black and 8 white marble, the other 8 black and 7 white marble. A box is selected at random (each choice is equally likely), and a marble is drawn from it at random (each choice is equally likely).

What is the probability that the **first box was the one selected** given that the **marble is black**?

Answer = number (3 significant figures)

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Sol F = first box is selected.

B = Black ball is drawn.

$$\mathbb{P}(F \mid B) = \frac{\mathbb{P}(B \mid F)\,\mathbb{P}(F)}{\mathbb{P}(B)}$$

$$= \frac{\mathbb{P}(B \mid F)\,\mathbb{P}(F)}{\mathbb{P}(B \mid F)\,\mathbb{P}(F) + \mathbb{P}(B \mid F^c)\,\mathbb{P}(F^c)}$$

$$= \frac{\frac{10}{18}\cdot\frac{1}{2}}{\frac{10}{18}\cdot\frac{1}{2} + \frac{8}{15}\cdot\frac{1}{2}} = \frac{\frac{5}{9}}{\frac{5}{9} + \frac{8}{15}} = \frac{25}{25+24}$$

$$= \frac{25}{49}$$

Random Voter

A total of 36 percent of the voters in a certain city classify themselves as Independents, whereas 36 percent classify themselves as Liberals and 28 percent say that they are Conservatives.

In a recent local election, 68 percent of the Independents, 63 percent of the Liberals, and 79 percent of the Conservatives voted. A voter is chosen at random.

Given that this person voted in the local election, what is the probability that he or she is a Conservative?

Answer = number (3 significant figures)

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Sol V = the person voted

C, L, I = Conservatives, Liberals, Independents

$$P(C \mid V) = \frac{P(V \mid C)\,P(C)}{P(V \mid C)\,P(C) + P(V \mid L)\,P(L) + P(V \mid I)\,P(I)}$$

$$= \frac{(0.79)\cdot(0.28)}{(0.79)\cdot(0.28) + (0.63)\cdot(0.36) + (0.68)\cdot(0.36)}$$

$$\approx 0.31928$$

Probability of Winning

9 distinct numbers are randomly distributed to players numbered 1 through 9. Whenever two players compare their numbers, the one with the higher one is declared the winner. Initially, players 1 and 2 compare their numbers; the winner then compares her number with that of player 3, and so on.

Let X denote the number of times player 2 is a winner. Find P$(X = 8)$.

Answer = number (3 significant figures)

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<u>Sol</u> Let X be the number of times the i-th player is a winner, for $i = 1, \cdots, 9$.

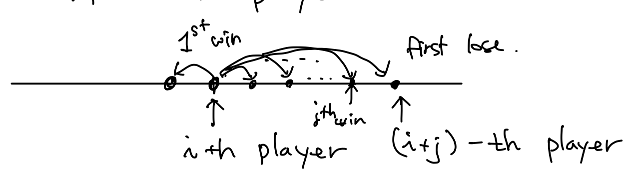
We want to find $\mathbb{P}(X = j)$ for $j = 1, \cdots, \min\{8, 10-i\}$

<u>Case 1</u>: The player never lose. (i.e. $\begin{cases} i \geq 2 \\ i+j = 10 \end{cases}$ or $\begin{cases} i=1 \\ j=8 \end{cases}$)

In this case, the player should have the biggest number.

Thus, $\mathbb{P}(X = j) = \frac{1}{9}$.

<u>Case 2</u>: If the player lose,



$\Rightarrow$ $(i+j)$-th player has the largest number among the first $(i+j)$ players

i-th player has the 2^{nd} largest number among first $(i+j)$ players

Choose $(i+j)$ numbers and the largest two numbers go to ith & $(i+j)$-th players.

$$\Rightarrow \mathbb{P}(X = j) = \frac{\boxed{\binom{9}{i+j} \cdot (i+j-2)!} \cdot (9-(i+j))!}{9!} = \frac{1}{(i+j)(i+j-1)}$$