4/18/22

Recall

- $\mathrm{Cov}(X,Y) = \mathbb{E}[XY] - \mathbb{E}X \cdot \mathbb{E}Y$
$$= \mathbb{E}[(X-\mathbb{E}X)(Y-\mathbb{E}Y)]$$

- $\mathrm{Cov}(X,Y) = \mathrm{Cov}(Y,X)$, $\mathrm{Cov}(X,X) = \mathrm{Var}(X)$

$$\mathrm{Cov}(aX+b,Y) = a\,\mathrm{Cov}(X,Y)$$

$$\mathrm{Cov}(X_1+\cdots+X_n, Y_1+\cdots+Y_m) = \sum_{i=1}^{n}\sum_{j=1}^{m} \mathrm{Cov}(X_i,Y_j)$$

Correlation Coefficient

$$\rho(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}}$$

Properties

(i) $-1 \leq \rho(X,Y) \leq 1$

(ii) $\rho(aX+b, Y) = \rho(X,Y)$

(iii) $|\rho(X,Y)| = 1 \iff Y = aX+b$

$\left(\iff \mathbb{E}[(\bar{Y} - t\bar{X})^2] = 0 \text{ for some } t \in \mathbb{R} \right)$

Example Toss a fair coin 3 times.

$X = \#$ of heads, $Y = \#$ of Tails $= 3-X$

$$\mathrm{Cov}(X,Y) = \mathrm{Cov}(X, 3-X) = -\mathrm{Var}(X)$$
$$= -\sum \mathrm{Var}(X_i) = -3\cdot\tfrac{1}{2}\cdot\tfrac{1}{2} = -\tfrac{3}{4}.$$