Quiz 5 Solution

MATH 461 S22

Sum of Binomials

If $X \sim \text{Bin}(n,p)$ and $Y \sim \text{Bin}(n,p)$ are independent, then $X+Y \sim \text{Bin}(n,2p)$.

- (a) True
- (b) False

Save & Grade | Save only | New variant

Sum of Binomials

If $X \sim \text{Bin}(n,p)$ and $Y \sim \text{Bin}(n,p)$ are independent, then $X+Y \sim \text{Bin}(2n,p)$.

- (a) False
- (b) True

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- If $X \sim \text{Bin}(n,p)$, $Y \sim \text{Bin}(m,p)$, and they are independent, then $X+Y \sim \text{Bin}(n+m,p)$.
- If $X \sim \text{Bin}(n,p)$, $Y \sim \text{Bin}(n,q)$ and $p \neq q$ then the sum may not be a binomial.

Independent Random Variabls

Let X, Y be jointly distributed continuous random variables with joint density given by

$$f(x,y) = \begin{cases} 4(x - xy) & 0 < x < 1, 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then X and Y are

- (a) independent.
- (b) dependent.

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$$f(x,y) = 4(x - xy) \cdot \mathbb{1}_{(0,1)}(x) \cdot \mathbb{1}_{(0,1)}(y)$$

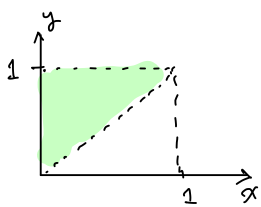
$$= \underbrace{\left(2x \cdot \mathbb{1}_{(0,1)}(x)\right)}_{\text{a function of } x} \cdot \underbrace{\left(2(1-y) \cdot \mathbb{1}_{(0,1)}(y)\right)}_{\text{a function of } y}$$

$\therefore$ Independent.

Independent Random Variabls

Let X, Y be jointly distributed continuous random variables with joint density given by

$$f(x,y) = \begin{cases} 8xy & 0 < x < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$



Then X and Y are

- (a) dependent.
- (b) independent.

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Since $f(x,y) \neq g(x) \cdot h(x)$ (due to the bound $0<x<y<1$), they are dependent. In fact,

$$f_X(x) = \int_x^1 8xy \, dy = 4x \cdot (1 - x^2), \quad f_Y(y) = \int_0^y 8xy \, dx = 4y^3$$

and $f(x,y) \neq f_X(x) \cdot f_Y(y)$.

Independent Random Variabls

Let X, Y be jointly distributed continuous random variables with joint density given by

$$f(x,y) = \begin{cases} 3(x - x^2 y) & 0 < x < 1, 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then X and Y are

- (a) dependent.
- (b) independent.

Save & Grade | Save only | New variant

$f(x,y) \neq g(x)\cdot h(y) \quad \therefore$ Dependent. In fact,

$f_X(x) = \int_0^1 3x(1 - xy)\,dy = 3x\cdot(1 - \frac{1}{2}x)$ if $0 < x < 1$ o.w 0

$f_Y(y) = \int_0^1 (3x - 3x^2 y)\,dx = \frac{3}{2} - y$ if $0 < y < 1$ o.w 0

$f(x,y) \neq f_X(x)\cdot f_Y(y)$.

Sum of Uniform RVs

Let X and Y be independent continuous random variables with $X \sim \text{Uniform}(0, 12)$ and $Y \sim \text{Uniform}(0, 6)$.

Compute the probability $\mathbb{P}(3 \leq X + Y \leq 4)$.

Answer = number (3 significant figures)

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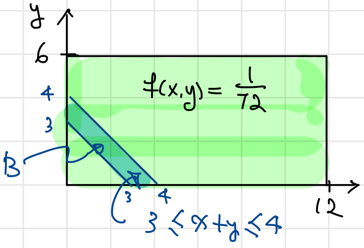

$f(x,y) = 0$

$P(3 \leq X + Y \leq 4)$

$= \iint_B \underbrace{\frac{1}{72}}_{f(x,y)}\,dx\,dy$ Th

$= \frac{1}{72}\cdot\frac{1}{2}(4^2 - 3^2) = \frac{7}{144}$