Homework 9

Math 461: Probability Theory, Spring 2022
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Due date: Apr 8, 2022

Instruction

1. Each problem is worth 10 points and only five randomly chosen problems will be graded.
2. Convert a photocopy of your solutions to **one single pdf file** and upload it on Moodle.
3. Please indicate whom you worked with, it will not affect your grade in any way.

1. If X_1, X_2, X_3 are independent random variables that are uniformly distributed over $(0, 1)$, compute the probability that the largest of the three is greater than the sum of the other two.

2. An ambulance travels back and forth at a constant speed along a road of length L. At a certain moment of time, an accident occurs at a point uniformly distributed on the road. (That is, the distance of the point from one of the fixed ends of the road is uniformly distributed over $(0, L)$.) Assuming that the ambulance's location at the moment of the accident is also uniformly distributed, and assuming independence of the variables, compute the probability density function and mean of the distance of the ambulance from the accident.

3. Let X be uniform on $(0, 1)$, and let Y be exponential with $\lambda = 1$ and let X, Y be independent.
(a) Find the pdf of $U = X + Y$.
(b) Also find the pdf of $V = X/Y$.

4. The gross weekly sales at a certain restaurant is a normal random variable with mean \$2200 and standard deviation \$230. What is the probability that
(a) the total gross sales over the next 2 weeks exceeds \$5000;
(b) weekly sales exceed \$2000 in at least 2 of the next 3 weeks?

5. The monthly worldwide average number of airplane crashes of commercial airlines is 2.2. What is the probability that there will be
(a) more than 2 such accidents in the next month?
(b) more than 4 such accidents in the next 2 months?
(c) more than 5 such accidents in the next 3 months?
Explain your reasoning!

6. Choose a number X at random from the set of numbers $\{1, 2, 3, 4, 5\}$. Now choose a number at random from the subset no larger than X, that is, from $\{1, \ldots, X\}$. Call this second number Y.
(a) Find the joint mass function of X and Y.
(b) Find the conditional mass function of X given that $Y = i$. Do it for $i = 1, 2, 3, 4, 5$.
(c) Are X and Y independent? Why?

7. The joint probability mass function of X and Y is given by

$p(i,j)$	$j = 1$	$j = 2$
$i = 1$	$\frac{1}{8}$	$\frac{1}{4}$
$i = 2$	$\frac{1}{8}$	$\frac{1}{2}$

(a) Compute the conditional mass function of X given $Y = j, j = 1, 2$.
(b) Are X and Y independent?
(c) Compute $\mathbb{P}(XY \leqslant 3), \mathbb{P}(X + Y > 2), \mathbb{P}(X/Y > 1)$.

8. The joint density function of X and Y is given by

$$f(x, y) = 2xe^{-x(y+2)}, \ x > 0, y > 0.$$

(a) Find the conditional density of X, given $Y = y$, and that of Y, given $X = x$.
(b) Find the density function of $Z = XY$.

9. The joint density function of X and Y is given by

$$f(x, y) = c(x^2 - y^2)e^{-x}, 0 \leqslant x < \infty, -x \leqslant y \leqslant x.$$

Find the conditional distribution of Y, given $X = x$.

10. If $X_1, X_2, \ldots, X_6$ are independent and identically distributed exponential random variables with the parameter λ, compute (a) $\mathbb{P}(\min(X_1, X_2, \ldots, X_6) \leqslant a)$ and (b) $\mathbb{P}(\max(X_1, X_2, \ldots, X_6) \leqslant a)$.