

Math 1554 Linear Algebra Spring 2023

Midterm 1

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

Name: _____ GTID Number: _____

Student GT Email Address: _____@gatech.edu

Section Number (e.g. A3, G2, etc.) _____ TA Name _____

Circle your instructor:

Prof Kim Prof Barone Prof David/Schroeder Prof Kumar

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- **Simplify your answers** unless explicitly stated otherwise.
- **Fill in circles** completely. Do not use check marks, X's, or any other marks.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- The last page is for scratch work. Please use it if you need extra space.
- This exam has 8 pages of questions.

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You do not need to justify your reasoning for questions on this page.

1. (a) (8 points) Suppose A is an $m \times n$ matrix and $\vec{b} \in \mathbb{R}^m$ unless otherwise stated. Select **true** if the statement is true for all choices of A and \vec{b} . Otherwise, select **false**.

true false

 If a vector \vec{b} can be written **uniquely** as a linear combination of vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ then there is a pivot in the first three columns of the matrix $(\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3 \ \vec{b})$.

 If $A\vec{x} = \vec{b}$ is consistent, then \vec{b} is in the span of the columns of A .

$$A\vec{v} = \vec{b} \quad , \quad A\vec{w} = \vec{b}$$

 If \vec{v} and \vec{w} are solutions to an inhomogeneous system $A\vec{x} = \vec{b}$, then $\vec{v} - \vec{w}$ is a solution to $A\vec{x} = \vec{0}$. Need: $A \cdot (\vec{v} - \vec{w}) = \vec{0}$ ✓

$$A \cdot (\vec{v} - \vec{w}) = A\vec{v} - A\vec{w} = \vec{b} - \vec{b} = \vec{0}$$

 If $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is linearly dependent, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent.

RRR +

 If A is size 3×4 and none of the rows of A consist entirely of zeros, then A has 3 pivots.

 If A and B are square $n \times n$ matrices, then $A^2 - B^2 = (A - B)(A + B)$.

 If A is size $m \times n$ with $m \neq n$ and the columns of A are linearly independent, then the transformation $T(\vec{x}) = A\vec{x}$ is onto.

 If the coefficient matrix A for a system of linear equations has a pivot in every row, then the system $A\vec{x} = \vec{b}$ has a solution for any \vec{b} in \mathbb{R}^m .

$$A\vec{x} = \vec{b} \quad \text{inconsistent.}$$

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(b) (4 points) Indicate whether the following situations are possible or impossible.

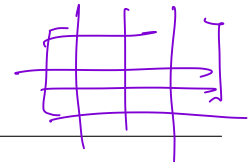
possible impossible

 An $m \times n$ matrix A with a pivot in its last column such that $A\vec{x} = \vec{b}$ is inconsistent.

 Two nonzero vectors \vec{v}_1, \vec{v}_2 such that $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent and $\{\vec{v}_1 - \vec{v}_2, \vec{v}_1 + \vec{v}_2\}$ is linearly dependent.

 A matrix A of size 4×3 with linearly dependent columns.

 A transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ that is onto.



$$\{v_1 - v_2, v_1 + v_2\}$$

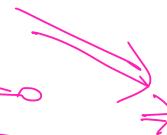
lin. indep. \Leftrightarrow

$$a_1(\vec{v}_1 - \vec{v}_2) + a_2(\vec{v}_1 + \vec{v}_2) = \vec{0}$$

$$(a_1 + a_2)\vec{v}_1 + (-a_1 + a_2)\vec{v}_2 = \vec{0}$$

\Rightarrow

$$\begin{cases} a_1 + a_2 = 0 \\ -a_1 + a_2 = 0 \end{cases}$$



$$\begin{cases} a_1 = 0 \\ a_2 = 0 \end{cases}$$

implies $c_1 = c_2 = 0$

(c) (2 points) If A is an $m \times 5$ matrix and $A\vec{x} = 0$ has a unique solution, then which of the following is true. Select only one.

- $m \geq 5$
- $m = 5$
- $m \leq 5$
- m can be any natural number

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(d) (3 points) For the vectors

$$\vec{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \vec{v}_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$$

Which of the following sets are linearly independent? Select all that apply.

- $\{\vec{v}_1, \vec{v}_2\}$
- $\{\vec{v}_2, \vec{v}_3\}$
- $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$

$T \Rightarrow A = [\tau(e_1) \quad \tau(e_2) \quad \tau(e_3)]$

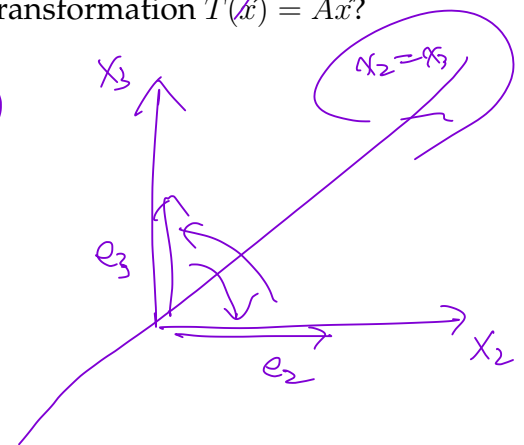
(e) (2 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Which of following accurately describes the transformation $T(\vec{x}) = A\vec{x}$?

Select only one.

- Rotation by $\frac{\pi}{2}$ radians around the x axis.
- Rotation by $\frac{\pi}{2}$ radians around the z axis.
- Reflection across the $x = 0$ plane.
- Reflection across the $y = z$ plane.



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

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2. (2 points) If possible, fill in the box with the missing element of the vector \vec{w} with a number so that $\{\vec{u}, \vec{v}, \vec{w}\}$ is linearly dependent. If it is not possible write NP in the space.

$$\vec{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} 5 \\ \square \\ 3 \end{pmatrix}$$

3. (4 points) Find b and c such that $AB = BA$.

$$A = \begin{pmatrix} 2 & b \\ -3 & c \end{pmatrix} \quad B = \begin{pmatrix} 4 & -5 \\ 3 & 5 \end{pmatrix}$$

$$b = \square \quad c = \square$$

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You do not need to justify your reasoning for questions on this page.

4. (8 points) Let T be the linear transformation defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ 3x_2 \\ 2x_1 - 4x_2 \end{bmatrix}.$$

(i) What is domain and codomain of T ?

domain is
codomain is

(ii) What is the standard matrix of T ?

(iii) Find a vector \vec{x} such that $T(\vec{x}) = \vec{b}$, where $\vec{b} = \begin{bmatrix} -2 \\ 9 \\ -4 \end{bmatrix}$.

 $\vec{x} =$

(iv) Is T one-to-one? yes no

(v) Is T onto? yes no

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5. (5 points) **Show all work for problems on this page.**

For what value(s) of h is the following set of vectors linearly dependent?

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ h \\ h^2 \end{pmatrix} \right\}$$

$$h = \boxed{}$$

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6. (5 points) **Show your work in the space below and put your answer in the box.** Provide a *dependence relation* on $\vec{v}_1, \vec{v}_2, \vec{v}_3$, i.e., a nontrivial linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_3$ equaling the zero vector which demonstrates that the vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are linearly dependent.

For full credit, show how the dependence relation is obtained by row reducing the appropriate coefficient matrix.

You may leave your answer in terms of $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

$$\vec{v}_1 = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix}, \quad \vec{v}_3 = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$$

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7. Show your work in the space below the first box and put your answers in the boxes.

- (a) (5 points) Write the parametric vector form for the general solution to the inhomogeneous equation $A\vec{x} = \vec{b}$.

$$A = \begin{pmatrix} 2 & 1 & 0 & -4 & 1 \\ 5 & 3 & 0 & -10 & 4 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 7 \\ 18 \end{pmatrix}$$

- (b) (2 points) For the homogeneous system with the same coefficient matrix A as part (a) above, write down the general solution to $A\vec{x} = \vec{0}$. *Hint: use your answer from part (a).*

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Circle your instructor:

Prof Vilaca Da Rocha Prof Kafer Prof Barone Prof Wheeler
Prof Blumenthal Prof Sun Prof Shirani

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1. (a) (8 points) Suppose A is an $m \times n$ matrix and $\vec{b} \in \mathbb{R}^m$ unless otherwise stated. Select **true** if the statement is true for all choices of A and \vec{b} . Otherwise, select **false**.

true false

- If A has a pivot in every column then the system $A\vec{x} = \vec{b}$ has a unique solution.
- Suppose A is a 6×4 matrix with 4 pivots, then there is \vec{b} such that $A\vec{x} = \vec{b}$ has no solution.
- The sets $\{\vec{v}_1, \vec{v}_2\}$ and $\{\vec{v}_1 + \vec{v}_2, -\vec{v}_1 - \vec{v}_2\}$ have the same span.
- If A and B are square $n \times n$ matrices, then $A^2 - B^2 = (A - B)(A + B)$.
- The matrix equation $A\vec{x} = \vec{0}$ is always consistent.
- Suppose $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are nonzero vectors in \mathbb{R}^n and the sets $\{\vec{v}_1, \vec{v}_2\}$, $\{\vec{v}_1, \vec{v}_3\}$, and $\{\vec{v}_2, \vec{v}_3\}$ are all linearly independent. Then, $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.
- If $A\vec{v} = 0$, $A\vec{u} = 0$ and $\vec{w} = 3\vec{v} - 2\vec{u}$, then $A\vec{w} = 0$. ↑ T is onto.
- Let $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation such that $T(\vec{x}) = \vec{b}$ has a solution for every $\vec{b} \in \mathbb{R}^m$. Then T is one-to-one. ↑
-

- (b) (4 points) Indicate whether the following situations are possible or impossible.

possible impossible

- A 7×5 matrix A with linearly independent columns.
- A linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ that is not onto and its standard matrix has linearly independent columns.
- $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ that is onto and its standard matrix has exactly one non-pivotal column.
- Two non-zero matrices A, B of size 2×2 with $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.
-

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You do not need to justify your reasoning for questions on this page.

(c) (2 points) Let

$$\left(\begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & 3h & 3 & 6 \\ 0 & 0 & 1 & 2 \end{array} \right)$$

be an augmented matrix of a system of linear equations. For which values of h does the system have a free variable? Choose the best option.

- 0 only
- $\frac{1}{3}$ only
- 1 only
- for all values of h
- for no values of h

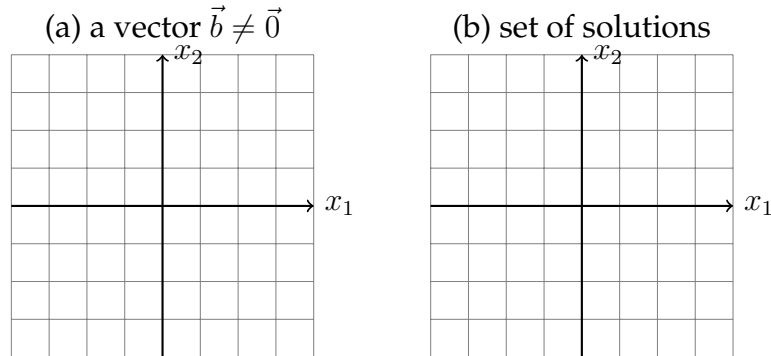
(d) (2 points) A linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^1$ maps each of the standard unit vectors \vec{e}_1 , \vec{e}_2 and \vec{e}_3 to 1. Which of the following statements is TRUE? Select only one.

- T is one-to-one.
- T is not onto.
- The solution set of $T(\vec{x}) = \vec{0}$ spans a plane in \mathbb{R}^3 .
- The range of T is $\{1\}$.

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2. (4 points) Suppose $A = \begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}$ and sketch (a) a non-zero vector \vec{b} such that $A\vec{x} = \vec{b}$ is consistent, and (b) the set of solutions to $A\vec{x} = \vec{0}$.



3. (2 points) Consider the linear system in variables x_1, x_2, x_3 with unknown constants below.

$$a_1x_1 + a_2x_2 + a_3x_3 = b_1$$

$$c_1x_1 + c_2x_2 + c_3x_3 = b_2$$

Which of the following statements about the solution set of this system are possible?
Select all that apply.

- The solution set is empty.
- The solution set is a single point.
- The solution set is a line.
- The solution set is a plane.

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4. Fill in the blanks.

- (a) (3 points) Let A be a coefficient matrix of size 2×2 and B be a coefficient matrix of size 3×2 . Construct an example of two augmented matrices $[A|\vec{b}]$ and $[B|\vec{d}]$ which are both in RREF and such that the systems $A\vec{x} = \vec{b}$ and $B\vec{x} = \vec{d}$ each have the exact same unique solution $x_1 = 3$ and $x_2 = 6$. If this is not possible write NP in each box.

$$[A|\vec{b}] = \boxed{\phantom{\begin{matrix} & & & & \\ & & & & \\ & & & & \end{matrix}}}$$

$$[B|\vec{d}] = \boxed{\phantom{\begin{matrix} & & & & \\ & & & & \\ & & & & \end{matrix}}}$$

- (b) (2 points) Let $\vec{u}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\vec{u}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and $\vec{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Find c_1, c_2 such that $\vec{b} = c_1\vec{u}_1 + c_2\vec{u}_2$.

$$c_1 = \boxed{} \quad c_2 = \boxed{}$$

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5. (8 points) Let T be a linear transformation that maps \vec{v}_1 to $T(\vec{v}_1)$ and \vec{v}_2 to $T(\vec{v}_2)$, where

$$\vec{v}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \quad T(\vec{v}_1) = \begin{pmatrix} 1 \\ 3 \\ 0 \\ 1 \end{pmatrix}, \quad T(\vec{v}_2) = \begin{pmatrix} 3 \\ -1 \\ -2 \\ 1 \end{pmatrix}.$$

(i) What is domain and codomain of T ?

domain is

codomain is

(ii) Is it true that $\mathbb{R}^2 = \text{span}\{\vec{v}_1, \vec{v}_2\}$?

yes

no

(iii) Write $\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ as linear combinations of \vec{v}_1 and \vec{v}_2 .

$\vec{e}_1 =$

$\vec{e}_2 =$

(iv) What is the standard matrix of T ?

(v) Is T one-to-one?

yes

no

Midterm 1. Your initials: _____

6. Show all work for problems on this page.

(a) (3 points) For what value of k will matrix A have exactly two pivots?

$$A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 2 \\ 0 & 1 & k \end{pmatrix}$$

$$k = \boxed{}$$

(b) (4 points) Find b and c such that $AB = BA$.

$$A = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & b \\ c & 0 \end{pmatrix}$$

$$b = \boxed{} \quad c = \boxed{}$$

Midterm 1. Your initials: _____

7. (4 points) **Show your work for problems on this page.**

Write down the parametric vector form for solutions to the homogeneous equation $A\vec{x} = \vec{0}$.

$$A = \begin{bmatrix} 1 & -1 & -2 & -3 & -1 \\ 0 & 1 & 0 & 3 & 1 \\ -1 & 1 & 2 & 3 & 2 \end{bmatrix}$$

8. (4 points) Determine whether the set of vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent. Justify your answer in the space below.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ -1 \\ 8 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} -2 \\ 2 \\ -9 \end{bmatrix}$$

linearly independent linearly dependent

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Circle your instructor:

Prof Barone Prof Shirani Prof Simone Prof Timko

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1. (a) (8 points) Suppose A is an $m \times n$ matrix and $\vec{b} \in \mathbb{R}^m$ unless otherwise stated. Select **true** if the statement is true for all choices of A and \vec{b} . Otherwise, select **false**.

true false

- The span of two non-zero vectors in \mathbb{R}^3 is necessarily a plane.
- If an echelon form of A has a row of zeros, then the system $A\vec{x} = \vec{b}$ has a free variable.
- If $A\vec{v} = \vec{b}$, and $[A \mid \vec{b}]$ is row equivalent to $[C \mid \vec{d}]$, then $C\vec{v} = \vec{d}$.
- If the columns of A span \mathbb{R}^m , then $A\vec{x} = \vec{b}$ is consistent for any $\vec{b} \in \mathbb{R}^m$.
- If $A\vec{x} = \vec{0}$ has a non-trivial solution, then the columns of A are linearly dependent.
- If \vec{v} and \vec{u} are solutions of a homogeneous system of linear equations, then $\vec{v} + \vec{u}$ is also a solution of that system.
- If the columns of a matrix A are linearly dependent, then the system $A\vec{x} = \vec{b}$ can not have a unique solution.
- If $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a linear transformation such that $T(\vec{x}) = \vec{b}$ has no solution for some $\vec{b} \in \mathbb{R}^n$, then T is not one-to-one.
-

- (b) (4 points) Indicate whether the following situations are possible or impossible.

possible impossible

- A linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ that is not onto.
- A linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ that is onto and its standard matrix has two non-pivotal columns.
- A linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$ that is onto and its standard matrix has linearly dependent columns.
- Three non-zero matrices A, B, C of size 2×2 with $AC = BC$ and $A \neq B$.
-

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You do not need to justify your reasoning for questions on this page.

(c) (2 points) Let

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & h^2 & 0 \end{array} \right]$$

be a row echelon form of an augmented matrix of a system of linear equations. For which values of h is the system consistent? Choose the best option.

- for all values of h
- 0 only
- for no values of h
- 1 and -1 only

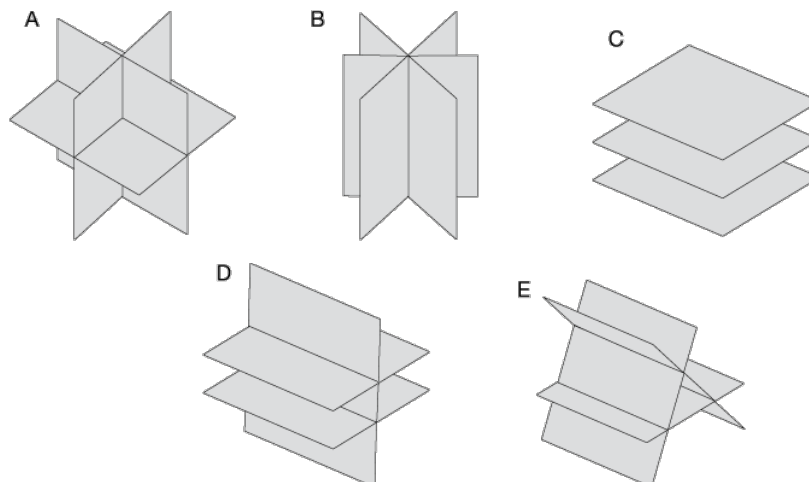
(d) (2 points) Let

$$\left[\begin{array}{ccccc|c} 1 & -1 & 0 & \pi & 2 & -1 \\ 0 & 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

be a row echelon form of an augmented matrix of a system of equations. How many solutions does the system have? Choose the best option.

- 0
- 1
- infinitely many

(e) (2 points) Suppose $A\vec{x} = \vec{b}$ is a system of three linear equations in three variables. If the system $A\vec{x} = \vec{b}$ is consistent, which of the following could be the graphs in \mathbb{R}^3 of the three equations represented by the rows of $[A | b]$? Circle all pictures that apply.

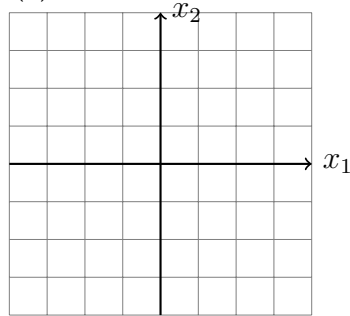


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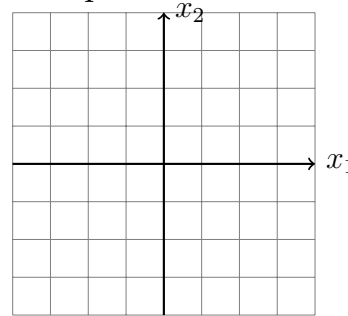
You do not need to justify your reasoning for questions on this page.

2. (4 points) Suppose $A = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$ and sketch a) a non-zero solution to $A\vec{x} = \vec{0}$, and b) the span of the columns of A .

(a) non-zero solution



(b) span of columns



3. (5 points) Let $A \in \mathbb{R}^{3 \times 5}$, $B \in \mathbb{R}^{4 \times 3}$ and $\vec{a} \in \mathbb{R}^3$, $\vec{b} \in \mathbb{R}^4$, $\vec{c} \in \mathbb{R}^5$. Which of the following are defined? Choose all the expressions which are defined.

$B\vec{b}$

$A\vec{c} \in \mathbb{R}^3$

$A(B\vec{a})$

$B(A\vec{c}) \in \mathbb{R}^4$

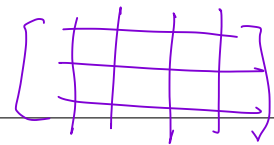
$B(\vec{a} + \vec{b})$

Handwritten notes for question 3:

- $A \in \mathbb{R}^{m \times n}$, $\vec{x} \in \mathbb{R}^n$, $\vec{b} \in \mathbb{R}^m$
- $A \cdot \vec{x} = \vec{b}$
- $A \cdot \vec{a} \in \mathbb{R}^3$
- $B \cdot \vec{a} \in \mathbb{R}^4$
- $A \cdot \vec{x} = \vec{0}$
- $A \cdot \vec{x} = \vec{b}$ (with $\vec{b} \neq \vec{0}$)

4. (3 points) In each of the following cases, indicate whether $A\vec{x} = \vec{b}$ has no solutions, a unique solution, infinitely many solutions, or if this can not be determined with the given information.

no solution	unique solution	infinitely many solutions	can't be determined
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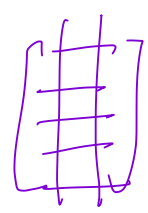


-

$A \in \mathbb{R}^{3 \times 4}$, $\vec{b} = \vec{0}$, and A has 2 pivots
homogeneous

-

$A \in \mathbb{R}^{5 \times 2}$, $\vec{b} = \vec{0}$, and A has 2 pivots



-

$A \in \mathbb{R}^{3 \times 5}$ and A has 3 pivots *onto*

for any \vec{b}

$A\vec{x} = \vec{b}$
consistent

1-1 : $T(\vec{x}) = \vec{0}$ implies $\vec{x} = \vec{0}$ / A Every column pivot
onto : For any $b \in \mathbb{R}^m$,
 $T(\vec{x}) = \vec{b}$ for some \vec{x} / Every row in A
pivot.

Midterm 1. Your initials: _____

You do not need to justify your reasoning for questions on this page.

5. Fill in the blanks.

(a) (2 points) If the augmented matrix $[A \mid \vec{b}]$ of a system of equations is 3×6 and the system has two pivot (basic) variables, then how many free variables does it have?

(b) (2 points) For what value(s) of h is the following set of vectors linearly dependent?

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ h \end{pmatrix}, \begin{pmatrix} 1 \\ h \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ h \end{pmatrix} \right\}$$

$$h = \boxed{}$$

Midterm 1. Your initials: _____

6. Show all work for problems on this page.

(a) (1 point) Let $\vec{b} = \begin{bmatrix} 3 \\ -4 \\ -6 \\ 1 \end{bmatrix}$, $\vec{a}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{a}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\vec{a}_3 = \begin{bmatrix} 3 \\ -3 \\ -5 \\ 2 \end{bmatrix}$. Is \vec{b} in the span of

$\vec{a}_1, \vec{a}_2,$ and \vec{a}_3 ?

Yes

No

(b) (2 points) If you answered yes to part (a), write \vec{b} as a linear combination of $\vec{a}_1, \vec{a}_2, \vec{a}_3$.
If you answered no, give an echelon form of the augmented matrix $[\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3 \ | \ \vec{b}]$.

Midterm 1. Your initials: _____

7. (3 points) Show your work for problems on this page.

Suppose that we have

$$\left[A \mid \vec{b} \right] \sim \left[\begin{array}{cccc|c} 1 & 4 & 0 & -1 & 3 \\ 0 & 0 & 1 & 5 & 2 \end{array} \right]$$

Find the parametric vector form for the solutions of $A\vec{x} = \vec{b}$.

||

RREF

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -4x_2 + x_4 + 3 \\ x_2 \\ -5x_4 + 2 \\ x_4 \end{bmatrix} = x_2 \begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ -5 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & 4 & 0 & -1 & 3 \\ 0 & 0 & 1 & 5 & 2 \end{array} \right] \rightarrow \begin{cases} x_1 + 4x_2 - x_4 = 3 \\ x_3 + 5x_4 = 2 \end{cases}$$
$$x_1 = -4x_2 + x_4 + 3$$
$$x_3 = -5x_4 + 2$$

8. (2 points) Suppose $A\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $A\vec{u} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$. Compute $A(2\vec{v} - \vec{u})$.



Midterm 1. Your initials: _____

9. (8 points) **Show all work for problems on this page.** Consider the linear transformation defined by $T(x_1, x_2) = (x_1 + x_2, x_1, x_1 - x_2)$ with domain \mathbb{R}^2 .

(i) What is the codomain of T ?

(ii) What is the standard matrix of T ?

(iii) Is T onto?

yes

no

(iv) Write an equation using the variables $b_1, b_2,$ and b_3 which is satisfied exactly when $T(x_1, x_2) = (b_1, b_2, b_3)$ has a solution for x_1, x_2 .

(v) What is the range of T ?