$\text{Span}\left\{\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}4\\8\end{pmatrix}\right\} = \left\{ a\begin{bmatrix}1\\2\end{bmatrix} + b\begin{bmatrix}4\\8\end{bmatrix} : a, b \in \mathbb{R}\right\} = \left\{ t\begin{bmatrix}1\\2\end{bmatrix} : t \in \mathbb{R}\right\}$

In-Class Midterm 1 Review, Math 1554

$A\vec{x} = \vec{b}_1$

$x_1 \cdot \vec{v} + 4x_2\vec{v} = -2\vec{v} \Rightarrow \underbrace{(x_1+4x_2+2)\vec{v}}_{} = 0$, $(x_1+4x_2+2) = 0$

1. Consider the matrix A and vectors $\vec{b}_1$ and $\vec{b}_2$.

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 8 \end{pmatrix}, \quad \vec{b}_1 = \begin{pmatrix} -2 \\ -4 \end{pmatrix}, \quad \vec{b}_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

v

If possible, on the grids below, draw

(i) the two vectors and the span of the columns of A,

(ii) the solution set of $A\vec{x} = \vec{b}_1$.

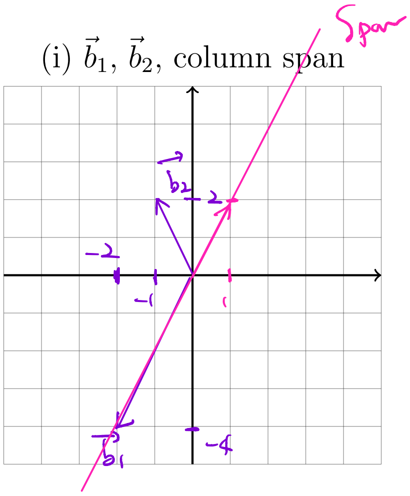
(iii) the solution set of $A\vec{x} = \vec{b}_2$.

$\vec{b}_1 \in \text{Span}$ $\Downarrow$ $A\vec{x} = \vec{b}_1$ is Consistent

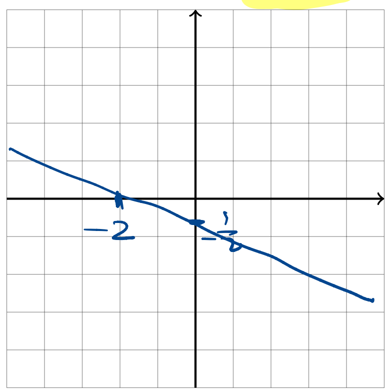
$\vec{b}_2 \notin \text{Span}$. $\Downarrow$ $A\vec{x} = \vec{b}_2$ Not Consistent

$x_1 + 4x_2 + 2 = 0$

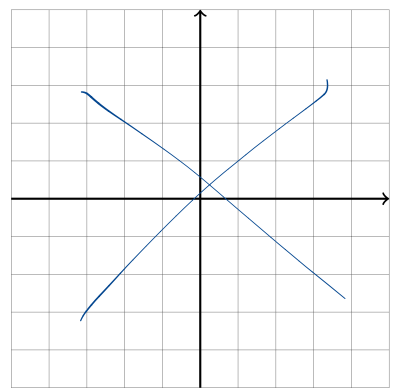
(i) $\vec{b}_1$, $\vec{b}_2$, column span



ii) solution set $Ax = \vec{b}_1$



iii) solution set $Ax = \vec{b}_2$



2. Indicate **true** if the statement is true, otherwise, indicate **false**. For the statements that are false, give a counterexample.

	true	false	counterexample
a) If $A \in \mathbb{R}^{M\times N}$ has linearly dependent columns, then the columns of A cannot span $\mathbb{R}^M$.	○	⊗	$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \end{bmatrix}$
b) If there are some vectors $\vec{b} \in \mathbb{R}^M$ that are not in the range of $T(\vec{x}) = A\vec{x}$, then there cannot be a pivot in every row of A.	⊗	○	
c) If the transform $\vec{x} \mapsto A\vec{x}$ projects points in $\mathbb{R}^2$ onto a line that passes through the origin, then the transform cannot be one-to-one.	○	○	

Col. of A span $\mathbb{R}^M$ $\Leftrightarrow$ $\text{Span}(\text{Cols}) = \mathbb{R}^M$

$\Leftrightarrow$ For any $\vec{b} \in \mathbb{R}^M$, $b = A \cdot \vec{x}$

$\Leftrightarrow$ " $\vec{b} \in \mathbb{R}^M$, $A\vec{x} = \vec{b}$ consistent

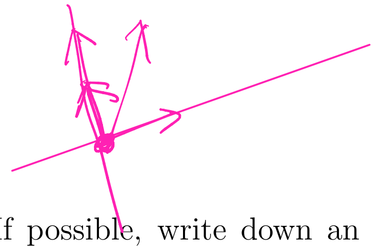
$T : \mathbb{R}^n \to \mathbb{R}^m$ is onto

1 - 1

$m \geq n$

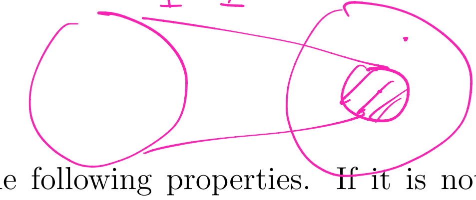
?

$n \geq m$



Not 1-1.

1-1



3. If possible, write down an example of a matrix with the following properties. If it is not possible to do so, write *not possible.*

   (a) A linear system that is homogeneous and has no solutions.

   NP.

   (b) A standard matrix A associated to a linear transform, T. Matrix A is in RREF, and $T_A : \mathbb{R}^3 \to \mathbb{R}^4$ is one-to-one.

   $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{4\times 3}$$

   (c) A 3×7 matrix A, in RREF, with exactly 2 pivot columns, such that $A\vec{x} = \vec{b}$ has exactly 5 free variables.

   $$\begin{array}{cc} 3 & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

4. Consider the linear system $A\vec{x} = \vec{b}$, where

$$A = \begin{pmatrix} 1 & 0 & 7 & 0 & -5 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$A\vec{x} = \vec{0}$

   (a) Express the augmented matrix $(\,A\,|\,\vec{b}\,)$ in RREF.

   $$\left[\begin{array}{ccccc|c} 1 & 0 & 7 & 0 & -5 & 1 \\ 0 & 1 & 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{array}\right] \xrightarrow[R_2 \to R_2 - R_3]{R_1 \to R_1 - 7R_3} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -5 & -13 \\ 0 & 1 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{array}\right]$$

   $$x_1 - 5x_5 = -13$$
   $$x_2 + 3x_5 = -2$$
   $$x_3 = 2$$

   (b) Write the set of solutions to $A\vec{x} = \vec{b}$ in parametric vector form. Your answer must be expressed as a vector equation.

   $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5x_5 - 13 \\ -3x_5 - 2 \\ 2 \\ x_4 \\ x_5 \end{bmatrix} = x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 5 \\ -3 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -13 \\ -2 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & h & 0 \\ h & 1 & h \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & h-1 & 1 \\ 0 & 1-h & 2h \end{bmatrix}$$

$h-1=0$ good

$\underline{h \neq 1}$

$$\longrightarrow \begin{bmatrix} 1 & 1 & \\ 0 & h-1 & 1 \\ 0 & 0 & 2h+1 \end{bmatrix}$$

$2h+1 = 0$

$h = 1, -\frac{1}{2}.$

$A = [1 \quad 1 \quad 1]$

$x_1 + x_2 + x_3 = 0$

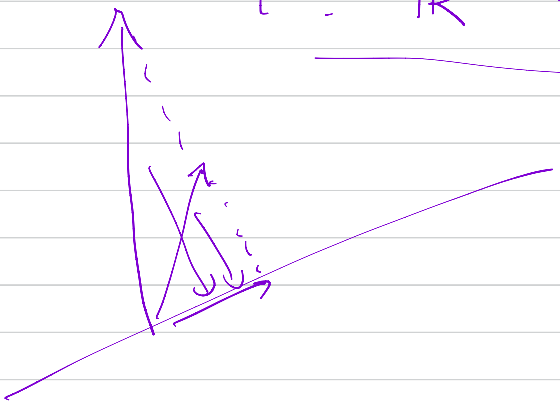
$\underline{T(x) = 0}$

$A\vec{x} = 0.$

Rang $= \mathbb{R}$.

Onto $\Longleftrightarrow$ Every Row Pivot

$T : \mathbb{R}^3 \to \mathbb{R}^2$



$$\begin{array}{cc} & 3 \\ 2 & \begin{bmatrix} 1 & 0 & \boxed{} \\ 0 & 1 & \boxed{} \end{bmatrix} \end{array} \begin{array}{l} \leftarrow \text{pivot} \\ \leftarrow \text{pivot} \end{array}$$

$$\begin{bmatrix} 1 & 0 & \underbrace{ & } \\ 0 & 1 & & \end{bmatrix}$$

$v_1, \dots, v_n \longrightarrow A = [v_1, \dots, v_n]$

$A\vec{x} = 0$ nontrivial solution

$\Rightarrow$ has a free variable