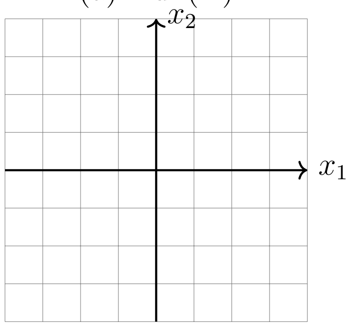
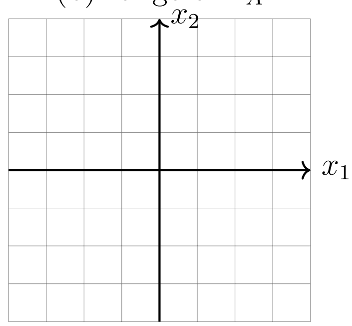
# Midterm 2 Lecture Review Activity, Math 1554

1. (3 points) $T_A$ is the linear transform $x \to Ax$, $A \in \mathbb{R}^{2\times 2}$, that projects points in $\mathbb{R}^2$ onto the $x_2$-axis. Sketch the nullspace of $A$, the range of the transform, and the column space of $A$. How are the range and column space related to each other?

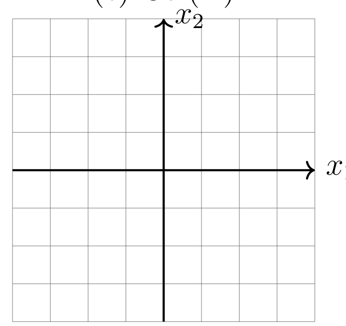
(a) Null($A$)




(b) range of $T_A$




(c) Col($A$)




2. Indicate **true** if the statement is true, otherwise, indicate **false**.

| | true | false |
|---|---|---|
| a) $S = \{\vec{x} \in \mathbb{R}^3 \mid x_1 = a, x_2 = 4a, x_3 = x_1x_2\}$ is a subspace for any $a \in \mathbb{R}$. $\begin{bmatrix} a \\ 4a \\ 4a^2 \end{bmatrix}$ $\neq 0$ | ○ | ⊗ |
| b) If $A$ is square and non-zero, and $A\vec{x} = A\vec{y}$ for some $\vec{x} \neq \vec{y}$, then $\det(A) \neq 0$. | ○ | ⊗ |

$\Rightarrow$ T Not 1-1 $\Rightarrow$ A Not invertible

3. If possible, write down an example of a matrix or quantity with the given properties. If it is not possible to do so, write *not possible*.

   (a) $A$ is $2 \times 2$, Col$A$ is spanned by the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\dim(\text{Null}(A)) = 1$. $A = \begin{pmatrix} 2 & 0 \\ 3 & 0 \end{pmatrix}$

   (b) $A$ is $2 \times 2$, Col$A$ is spanned by the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\dim(\text{Null}(A)) = 0$. $A = \begin{pmatrix} \text{NP.} \end{pmatrix}$

   (c) $A$ is in RREF and $T_A:\ \mathbb{R}^3 \to \mathbb{R}^3$. The vectors $u$ and $v$ are a basis for the range of $T$.

   $$u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, v = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, A = \begin{pmatrix} 1 & 0 & 100 \\ 0 & 1 & 1000 \\ 0 & 0 & 0 \end{pmatrix}$$

• $Ax = Ay$ $\quad A\underbrace{(x-y)}_{\neq 0} = 0$ has nontrivial sol.

4. Indicate whether the situations are possible or impossible by filling in the appropriate circle.

| | | possible | impossible |
|---|---|---|---|
| 4.i) | Vectors $\vec{u}$ and $\vec{v}$ are eigenvectors of square matrix $A$, and $\vec{w} = \vec{u}+\vec{v}$ is also an eigenvector of $A$. | ○ | ○ |
| 4.ii) | $T_A = A\vec{x}$ is one-to-one, $\dim(\mathrm{Col}(A)) = 4$, and $T_A: \mathbb{R}^3 \to \mathbb{R}^4$. | ○ | ○ |

5. (2 points) Fill in the blanks.

   (a) If $A$ is a $6 \times 4$ matrix in RREF and $\mathrm{rank}(A) = 4$, what is the rank of $A^T$? $\boxed{\phantom{xx}}$

   (b) $T_A = A\vec{x}$, where $A \in \mathbb{R}^{2\times 2}$, is a linear transform that first rotates vectors in $\mathbb{R}^2$ clockwise by $\pi$ radians about the origin, then scales their $x$-component by a factor of 3, then projects them onto the $x_1$-axis. What is the value of $\det(A)$? $\boxed{\phantom{xx}}$

6. (3 points) A virus is spreading in a lake. Every week,

   - 20% of the healthy fish get sick with the virus, while the other healthy fish remain healthy but could get sick at a later time.
   - 10% of the sick fish recover and can no longer get sick from the virus, 80% of the sick fish remain sick, and 10% of the sick fish die.

   Initially there are exactly 1000 fish in the lake.

   a) What is the stochastic matrix, $P$, for this situation? Is $P$ regular?

   b) Write down any steady-state vector for the corresponding Markov-chain.

S23 T/F

$$\begin{aligned} A\vec{x} &= \lambda\vec{x} \\ + \quad B\vec{x} &= \lambda\vec{x} \\ \hline (A+B)\vec{x} &= 2\lambda\,\vec{x} \end{aligned}$$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad 1 \qquad B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \quad 0$$

$$AB = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix} \qquad BA = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

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$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \qquad BA = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

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$AB\vec{x} = \vec{0}$ has nontrivial solution

$\Leftrightarrow$ $AB$ is NOT INVERTIBLE.

$\Leftrightarrow$ $\det(AB) = 0 = \det(A)\cdot\det(B)$

$$\Downarrow$$

$$\det(A) = 0$$

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0 eigenvalue $\Leftrightarrow$ $\det(A) = 0$

$\Leftrightarrow$ $T$ is Not onto

Fall 22 Makeup

$\dim(\mathrm{Null}(A)) = 1$

$$\begin{array}{ccc} 2P & & 1N \end{array}$$

$$A = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$Ax = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Fall 22

#9

$$\begin{array}{cccc} P & 1 & \frac{1}{2} & 0 \\ & \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} & \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \end{array}$$

$$\vec{x}_0 = \frac{1}{2}\vec{v}_1 + \frac{1}{2}\vec{v}_2$$

$$\vec{x}_3 = P^3 \cdot \left(\frac{1}{2}v_1 + \frac{1}{2}v_2\right) = \frac{1}{2} \cdot 1^3 \cdot v_1 + \frac{1}{2}\left(\frac{1}{2}\right)^3 \cdot v_2$$

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$$\vec{x}_0 = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ \frac{1}{4} \end{bmatrix} = \underline{c_1}\, v_1 + \underset{\sim}{c_2}\, v_2 + \underset{\sim}{c_3}\, v_3 = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & \frac{1}{4} \\ 1 & -1 & 1 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{4} \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & \frac{1}{4} \\ 0 & -1 & 2 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{4} \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{4} \\ 0 & -1 & 2 & \frac{1}{4} \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & -1 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{array}\right]$$

$$x_0 = \frac{1}{2} v_1 + \frac{1}{4} \cdot v_2 + \frac{1}{4} v_3$$

$$x_k = \boxed{\frac{1}{2} \cdot 1^k v_1} + \underbrace{\frac{1}{4} \cdot \left(\frac{1}{2}\right)^k v_2}_{\downarrow\, 0} + \underbrace{\frac{1}{4} \cdot (0)^k v_k}_{= 0}$$

$$= \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix}$$

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$$w = 1 \cdot v_1 - 3 \cdot v_2 + 4 v_3$$

$$Aw = 1 \cdot 1 \cdot v_1 - 3 \cdot (2) \cdot v_2 + 4 \cdot 0 \cdot v_3$$

$$AB\vec{x} = A \cdot (B\vec{x})$$
$$= A \cdot (\lambda \vec{x})$$
$$= \lambda \cdot (A\vec{x})$$
$$= \lambda \cdot \tilde{\lambda} \vec{x}$$