Math 1554 Linear Algebra Fall 2022

Midterm 3

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

Name: ________________________ GTID Number: ___________________

Student GT Email Address: ______________________________@gatech.edu

Section Number (e.g. A3, G2, etc.) _______ TA Name ________________________

Circle your instructor:

Prof Vilaca Da Rocha Prof Kafer Prof Barone Prof Wheeler
Prof Blumenthal Prof Sun Prof Shirani

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- **Simplify your answers** unless explicitly stated otherwise.
- **Fill in circles** completely. Do not use check marks, X's, or any other marks.
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- This exam has 7 pages of questions.

$\begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \cdot \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$ $D = \begin{pmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \ \lambda_n \end{pmatrix} \Rightarrow D^{-1} = \begin{pmatrix} \frac{1}{\lambda_1} & & \\ & \ddots & \\ & & \frac{1}{\lambda_n} \end{pmatrix}$

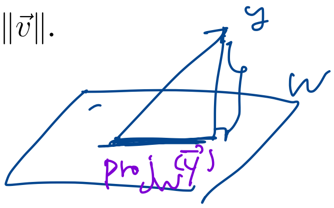
Midterm 3. Your initials: ______________

You do not need to justify your reasoning for questions on this page.

1. (a) (8 points) Suppose A is an $m \times n$ matrix and $\vec{b} \in \mathbb{R}^m$ unless otherwise stated. Select **true** if the statement is true for all choices of A and $\vec{b}$. Otherwise, select **false**.

$\phi_A(\lambda) = \det(A - \lambda I) = \det(A^T - \lambda I)$

true	false	
○	○	A matrix $A \in \mathbb{R}^{n\times n}$ and its transpose A^{T} have the same eigenvectors.
○	○	An invertible matrix A is diagonalizable if and only if its inverse A^{-1} is diagonalizable. $(A)^{-1} = (PDP^{-1})^{-1} = (P^{-1})^{-1} \cdot D^{-1} \cdot P^{-1} = P\underbrace{D^{-1}}_{\text{diagonal}}P^{-1}$
○	○	If $\vec{u}\cdot\vec{v} = \vec{u}\cdot\vec{w}$, then $\vec{u}$ is orthogonal to $(\vec{w}-\vec{v})$.
○	○	If the vectors $\vec{u}$ and $\vec{v}$ are orthogonal then $\|\vec{u}+\vec{v}\| = \|\vec{u}\| + \|\vec{v}\|$.
○	○	If $\vec{y} \in \mathbb{R}^n$ is a nonzero vector and W is a subspace of $\mathbb{R}^n$, then $\|$ ~~$\text{proj}_W(\vec{y})$~~ $\|$ is the shortest distance between W and $\vec{y}$. $\|y - \text{proj}_W(\vec{y})\|$
○	○	If $\vec{y} \in \mathbb{R}^n$ is a nonzero vector and W is a subspace of $\mathbb{R}^n$, then $\vec{y} - \text{proj}_W(\vec{y})$ is in $W^\perp$.
○	○	If W is a subspace of $\mathbb{R}^n$ and $\vec{y} \in \mathbb{R}^n$ such that $\vec{y}\cdot\vec{w} = 0$ for ~~some~~ vector $\vec{w} \in W$, then $\vec{y} \in W^\perp$.
○	○	The line of best fit $y = \beta_0 + \beta_1 x$ for the points $(1,2)$, $(1,3)$, and $(1,4)$ is unique.



(b) (4 points) Indicate whether the following situations are possible or impossible.

possible	impossible	
○	○	A 5×5 real matrix A such that A has no real eigenvalues.
○	○	An $m \times n$ matrix U where $U^T U = I_n$ and $n > m$.
○	○	A 2-dimensional subspace W of $\mathbb{R}^3$ and a vector $\vec{y} \in W$ such that $\|\vec{v}_1 - \vec{y}\| = \|\vec{v}_2 - \vec{y}\|$ where $\vec{v}_1, \vec{v}_2 \in W^\perp$ and $\vec{v}_1 \neq \vec{v}_2$.
○	○	A matrix $A \in \mathbb{R}^{3\times 4}$ such that the linear system $A\vec{x} = \vec{b}$ has a unique least-squares solution.

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You do not need to justify your reasoning for questions on this page.

3. (8 points) Fill in the blanks.

(a) Suppose $\vec{u}$ and $\vec{v}$ are orthogonal vectors in $\mathbb{R}^n$ and that $\vec{v}$ is a unit vector. If $(2\vec{u}+\vec{v})\cdot(\vec{u}+5\vec{v}) = 13$, determine the length of $\vec{u}$. $\|\vec{u}\| =$ ☐

(b) The normal equations for the least-squares solution to $A\vec{x} = \vec{b}$ are given by: ☐

(c) Compute the length (magnitude) of the vector $\vec{y}$.

$$\vec{y} = \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$\|\vec{y}\| =$ ☐

$$Av = \lambda v$$

(d) Let $A = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$. The vector $\vec{v} = \begin{pmatrix} -1-i \\ -1+i \end{pmatrix}$ is an eigenvector of A. Find the associated eigenvalue λ for the eigenvector $\vec{v}$ of A. $\lambda =$ ☐

$$\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} -1-i \\ -1+i \end{bmatrix} = \lambda \begin{bmatrix} -1-i \\ -1+i \end{bmatrix}$$

$$\begin{bmatrix} 2(-1-i) - 2(-1+i) \\ \ \end{bmatrix} = \begin{bmatrix} \lambda(-1-i) \\ \ \end{bmatrix}$$

$$-4i = \lambda(-1-i)$$

$$2+2i$$
$$\parallel$$

$$\lambda = \frac{-4i}{-1-i} = \frac{4i}{1+i} = \frac{4i(1-i)}{(1+i)(1-i)} = \frac{\overset{2}{\cancel{4}}i(1-i)}{\cancel{2}}$$

Math 1554 Linear Algebra Fall 2022

Midterm 3 Make-up

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Midterm 3 Make-up. Your initials: ______________

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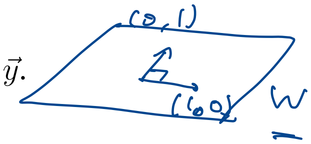
1. (a) (8 points) Suppose A is an $m \times n$ matrix and $\vec{b} \in \mathbb{R}^m$ unless otherwise stated. Select **true** if the statement is true for all choices of A and $\vec{b}$. Otherwise, select **false**.

true	false	
○	○	A matrix $A \in \mathbb{R}^{n\times n}$ and its transpose A^{T} have the same eigenvectors.
○	○	The line of best fit $y = \beta_0 + \beta_1 x$ for the points $(1,1), (2,1)$, and $(3,1)$ is unique.
○	○	If the vectors $\vec{u}$ and $\vec{v}$ are orthogonal then $\|\vec{u}+\vec{v}\| = \|\vec{u}\| + \|\vec{v}\|$.
○	○	A triangular matrix A is diagonalizable if and only if A is invertible.
○	○	If $A = PDP^{-1}$ where D is a diagonal matrix, then D and A have the same eigenvectors.
○	○	If $\vec{y} \in \mathbb{R}^n$ is a nonzero vector and W is a subspace of $\mathbb{R}^n$, then $\mathrm{proj}_W(\vec{y})$ is in W.
○	○	If $\vec{y} \in \mathbb{R}^n$ is a nonzero vector and W is a subspace of $\mathbb{R}^n$, then $\|\vec{y} - \mathrm{proj}_W(\vec{y})\|$ is the shortest distance between W and $\vec{y}$.
○	○	If W is a subspace of $\mathbb{R}^n$ and $\vec{y} \in \mathbb{R}^n$ such that $\vec{y}\cdot\vec{w} = 0$ for ~~some~~ all vector s $\vec{w} \in W$, then $\vec{y} \in W^{\perp}$. False

$y = (0,2)$

$w = (1,0)$



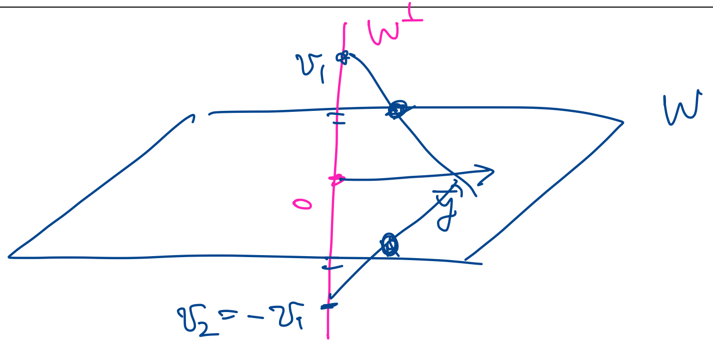
/ W 1-D , $\vec{w} \neq 0 \Rightarrow$ True)

(b) (4 points) Indicate whether the following situations are possible or impossible.

possible	impossible	
○	○	A 3×3 real matrix A such that A has eigenvalues $2, 3, 2i+3$.
○	○	An $m \times n$ matrix U where $U^TU = I_n$ and $n > m$. $\Rightarrow n \leq m$
☒	○	A 2-dimensional subspace W of $\mathbb{R}^3$ and a vector $\vec{y} \in W$ such that $\|\vec{v}_1 - \vec{y}\| = \|\vec{v}_2 - \vec{y}\|$ where $\vec{v}_1, \vec{v}_2 \in W^{\perp}$ and $\vec{v}_1 \neq \vec{v}_2$.
☒	○	A vector $\vec{y} \in \mathbb{R}^3$ and a subspace W in $\mathbb{R}^3$ such that $\vec{y} = \vec{w} + \vec{z}$ where $\vec{w}$ is in W, but $\vec{z}$ is not in $W^{\perp}$.

$\vec{z} = -\vec{W} \neq 0$

$\vec{y} = 0$



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7. (4 points) **Show all work for problems on this page.**
Let $\mathcal{B} = \{\vec{x}_1, \vec{x}_2, \vec{x}_3\}$ be a basis for a subspace W of $\mathbb{R}^4$, where

$$\vec{x}_1 = \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \qquad \vec{x}_2 = \begin{pmatrix} 1 \\ 2 \\ -2 \\ 1 \end{pmatrix}, \qquad \vec{x}_3 = \begin{pmatrix} -1 \\ -1 \\ 0 \\ 2 \end{pmatrix}.$$

(a) Apply the Gram-Schmidt process to the set of vectors $\{\vec{x}_1, \vec{x}_2, \vec{x}_3\}$ to find an orthogonal basis $\mathcal{H} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ for W. *Clearly show all steps of the Gram-Schmidt process.*

$$\vec{y}_1 = \vec{x}_1 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\vec{y}_2 = \vec{x}_2 - \text{proj}_{\vec{y}_1}(x_2)$$

$$= \vec{x}_2 - \frac{x_2 \cdot y_1}{y_1 \cdot y_1}\vec{y}_1$$

$$= \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} - \frac{\overset{1}{\cancel{(-1)}+\cancel{2+2}+\cancel{1}}}{\cancel{4}\ 1}\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\mathcal{H} = \boxed{\left\{ \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} \ \end{bmatrix} \right\}}$$

$$\vec{y}_3 = \vec{x}_3 - \frac{x_3 - y_1}{y_1 - y_1}\vec{y}_1 - \frac{x_3 \cdot y_2}{y_2 \cdot y_2}\vec{y}_2$$

$$= \begin{bmatrix} -1 \\ -1 \\ 0 \\ 2 \end{bmatrix} - \frac{2}{4}\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} - \frac{(-3)}{6}\begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} \vdots \end{bmatrix}$$

(b) In the space below, **check** that the vectors in the basis $\mathcal{H}$ form an orthogonal set.

$y_1 \cdot y_2 = 0 \qquad y_1 \cdot y_3 = 0 \qquad y_2 \cdot y_3 = 0$

Math 1554 Linear Algebra Spring 2022

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Circle your instructor:

Prof Barone Prof Shirani Prof Simone Prof Timko

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1. (a) (8 points) Suppose A is an $m \times n$ matrix and $\vec{b} \in \mathbb{R}^m$ unless otherwise stated. Select **true** if the statement is true for all choices of A and $\vec{b}$. Otherwise, select **false**.

true false

○ ○ The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ is diagonalizable.

○ ○ If W is the subspace of $\mathbb{R}^3$ spanned by $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$, then $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ is a vector in $W^\perp$.

○ ○ If U is a 3×2 matrix with orthonormal columns, then for every $\vec{y} \in \text{Col}(U)$ we have $\vec{y} = UU^T\vec{y}$.

○ ○ If the matrix A has orthogonal columns, then A^TA is a diagonal matrix.

○ ○ Assume $n \neq m$. If $A = QR$ is the QR factorization of $A \in \mathbb{R}^{n\times m}$, then $Q \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{n\times m}$.

○ ○ If $A = QR$ is a QR factorization of A, then $A^TA = R^TR$.

○ ○ If $\hat{x}$ and $\hat{y}$ are least-squares solutions of $A\vec{x} = \vec{b}$, then $\hat{x} - \hat{y} \in \text{Nul}(A)$.

○ ○ Suppose A is such that T_A is not one-to-one, and $\vec{b}$ is not in the range of T. Then $A\vec{x} = \vec{b}$ has a unique least-squares solution.

(b) (4 points) Indicate whether the following situations are possible or impossible.

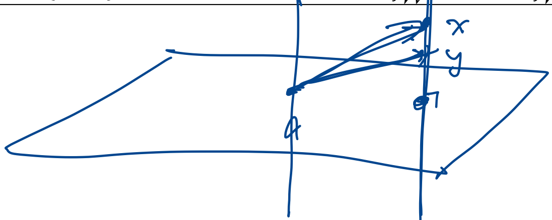
possible impossible

○ ○ A is a 7×7 diagonalizable matrix with exactly three distinct eigenvalues whose geometric multiplicities are 1, 2, and 3, respectively.

○ ○ $\vec{u}$ and $\vec{v}$ are nonzero vectors such that $||\vec{u} + \vec{v}||^2 = ||\vec{u}||^2 + ||\vec{v}||^2$.

○ ○ The distance between a vector $\vec{b} \in \mathbb{R}^m$ and the column space of a matrix $A \in \mathbb{R}^{m\times n}$ is zero, and the linear system $A\vec{x} = \vec{b}$ is inconsistent.

○ ○ $\mathcal{W}$ is a 2-dimensional subspace of $\mathbb{R}^3$, and there exists a linearly independent set of vectors $\{\vec{x}, \vec{y}\}$ in $\mathbb{R}^3$ such that $\text{Proj}_{\mathcal{W}}\vec{x} = \text{Proj}_{\mathcal{W}}\vec{y}$.



$W = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

$x = \begin{bmatrix} 1 \\ 1 \\ \textcircled{1} \end{bmatrix}$ $y = \begin{bmatrix} 1 \\ 1 \\ \textcircled{2} \end{bmatrix}$

orthogonal

$A \in \mathbb{R}^{3\times 3}$

You do not need to justify your reasoning for questions on this page.

(c) (2 points) The standard $\overset{A}{\text{matrix}}$ of a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ has orthonormal columns. Which one of the following statements is **false**?

Choose only one.

- ○ $\|T(\vec{x})\| = \|\vec{x}\|$ for all $\vec{x}$ in $\mathbb{R}^3$. True.
- ○ If two non-zero vectors $\vec{x}$ and $\vec{y}$ in $\mathbb{R}^3$ are scalar multiples of each other, then $\|T(\vec{x}+\vec{y})\|^2 = \|T(\vec{x})\|^2 + \|T(\vec{y})\|^2$. $\vec{x}=\vec{y}$
- ○ If $\mathcal{P}$ is a parallelpiped in $\mathbb{R}^3$, then the volume of $T(\mathcal{P})$ is equal to the volume of $\mathcal{P}$. T.
- ○ T is one-to-one.

$|\det(A)|\,\mathrm{Vol}(P) = \mathrm{Vol}(T(P))$

$T(\vec{x}) = A\vec{x}$

① $A^TA = I$

② $\|T(\vec{x})\|^2 = T(\vec{x}) \cdot T(\vec{x})$

$= (A\vec{x}) \cdot (A\vec{x})$

$= (A^T \cdot A \cdot \vec{x}) \cdot \vec{x} = \vec{x} \cdot \vec{x} = \|\vec{x}\|^2$

2. (2 points) Suppose that, in the QR factorization of A, we have Q as given below. Find R.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \qquad Q = \frac{1}{2}\begin{bmatrix} 1 & 1/\sqrt{3} \\ 1 & 1/\sqrt{3} \\ 1 & -\sqrt{3} \\ 1 & 1/\sqrt{3} \end{bmatrix}$$

Note: Please fill in the blanks and do not place values in front of the matrix for this problem.

$$R = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$$

A orthogonal $\iff$ $A^TA = I$

$$1 = \det(A^T \cdot A) = \underbrace{\det(A^T)}_{= \det(A)} \cdot \det(A) = \det(A)^2$$

$\det(A) = 1$ or -1

T is 1-1

$\iff$ $T(\vec{x}) = 0$ implies $\vec{x} = 0$

$\Rightarrow \|T(\vec{x})\| = 0 = \|\vec{x}\| \Rightarrow$

$\iff$ $A\vec{x} = 0$ has the only trivial solution.

$\iff$ A is invertible.

$\iff$ $\det(A) \neq 0$.

IMT.

Midterm 3. Your initials: _______________

You do not need to justify your reasoning for questions on this page.

3. (2 points) Using **only** 0's and 1's in your answer, give an example of a 2×2 matrix that is invertible but not diagonalizable.

$$\begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}$$

4. (6 points) Fill in the blanks.

   (a) Let $\vec{u}, \vec{v} \in \mathbb{R}^n$ be orthogonal vectors each with length 2. Determine the length of the vector $2\vec{u} + \vec{v}$. $\boxed{}$

   (b) Suppose A is a 7×5 matrix such that $\dim\,(\text{Row } A)^\perp = 4$. Determine the dimension of the column space of A. $\boxed{}$

   (c) Suppose $\mathcal{B} = \{\vec{v}_1, \vec{v}_2\}$ is an orthogonal basis for a subspace $\mathcal{W}$ of $\mathbb{R}^n$, and $\vec{x}$ belongs to the subspace $\mathcal{W}$. Suppose also that

$$\vec{v}_1 \cdot \vec{v}_1 = 2, \vec{v}_2 \cdot \vec{v}_2 = 4, \vec{v}_1 \cdot \vec{x} = 6, \text{ and } \vec{v}_2 \cdot \vec{x} = -4.$$

   Find $[\vec{x}]_{\mathcal{B}}$ the coordinates of $\vec{x}$ in the basis $\mathcal{B}$. $\boxed{\begin{bmatrix} 6/2 \\ -4/4 \end{bmatrix}}$

$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} a \\ b \end{bmatrix} \iff \vec{x} = a \cdot \vec{v}_1 + b \cdot \vec{v}_2$$

$$a = \boxed{\frac{\vec{x} \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1}} \qquad b = \frac{\vec{x} \cdot \vec{v}_2}{\vec{v}_2 \cdot \vec{v}_2}$$

$$\vec{x} \cdot \vec{v}_1 = a \cdot \vec{v}_1 \cdot \vec{v}_1$$

Midterm 3 Lecture Review Activity, Math 1554

1. Indicate **true** if the statement is true, otherwise, indicate **false**.

	true	false
a) If S is a two-dimensional subspace of $\mathbb{R}^{50}$, then the dimension of $S^\perp$ is 48.	●	○
b) An eigenspace is a subspace spanned by a single eigenvector.	○	●
c) The $n \times n$ zero matrix can be diagonalized.	●	○
d) A least-squares line that best fits the data points $(0, y_1), (1, y_2), (2, y_3)$ is unique for any values y_1, y_2, y_3.	●	○

2. If possible, give an example of the following.

2.1) A matrix, A, that is in echelon form, and $\dim\left(\overset{\text{Nul}(A)}{(\text{Row}A)^\perp}\right) = 2$, $\dim\left(\overset{\text{Nul}(A^T)}{(\text{Col}\,A)^\perp}\right) = 1$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

2.2) A singular 2×2 matrix whose eigenspace corresponding to eigenvalue $\lambda = 2$ is the line $x_1 = 2x_2$. The other eigenspace of the matrix is the x_2 axis.

$$\overset{\cancel{0}}{\boxed{\begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix}}} = \cancel{\tfrac{1}{2}}\begin{pmatrix} \cancel{4}2 & 0 \\ \cancel{x}1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\frac{1}{2}\begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$$

2.3) A subspace S, of $\mathbb{R}^4$, that satisfies $\dim(S) = \dim(S^\perp) = 3$.

N.P.

2.4) A 2×3 matrix, A, that is in RREF. $(\text{Row}\,A)^\perp$ is spanned by $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$.

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \end{pmatrix}$$

3. Circle **possible** if the set of conditions are create a situation that is possible, otherwise, circle **impossible**. For the situations that are possible give an example.

3.1) A is $n \times n$, $A\vec{x} = A\vec{y}$ for a particular $\vec{x} \neq \vec{y}$, $\vec{x}$ and $\vec{y}$ are in $\mathbb{R}^n$, and $\dim(\underbrace{(\text{Row } A)^{\perp}}_{\text{Nul}(A)}) \neq 0$.

[✓ **possible**] **impossible**

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$$

$\dim \text{Nul}(A^T - \lambda I))$

3.2) A is $n \times n$, $\lambda \in \mathbb{R}$ is an eigenvalue of A, and $\dim((\text{Col}(A - \lambda I))^{\perp}) = 0$.

possible [✓ **impossible**]

3.3) $\text{proj}_{\vec{v}}\vec{u} = \text{proj}_{\vec{u}}\vec{v}$, $\vec{v} \neq \vec{u}$, and $\vec{u} \neq \vec{0}$, $\vec{v} \neq \vec{0}$.

[✓ **possible**] **impossible**

$$\frac{u \cdot v}{u \cdot u} \cdot u = \frac{u \cdot v}{v \cdot v} \cdot v$$

if $u \cdot v = 0$

4. Consider the matrix A.

$$A = \begin{pmatrix} 1 & -3 & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Construct a basis for the following subspaces and state the dimension of each space.

4.1) $(\text{Row}A)^{\perp} = \text{Nul}(A)$ $\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 3 \\ 1 \end{bmatrix} \right\}$

4.2) Col A $= \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

4.3) $(\text{Col}A)^{\perp}$

$= \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$