Expectation of a Polynomial from mean and variance

Let X be a random variable with E$(X) = -4$ and Var$(X) = 2$. Find E$((-2X+6)^2)$.

Answer = number (3 significant figures)

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$\boxed{\text{Solution}}$ $Var(X) = 2 = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \mathbb{E}[X^2] - 16 \quad \therefore \mathbb{E}[X^2] = 18$

$$\begin{aligned}\mathbb{E}[(-2X+6)^2] &= 4\,\mathbb{E}[X^2] - 24\,\mathbb{E}[X] + 36 \\ &= 4 \cdot 18 - 24 \cdot (-4) + 36 \\ &= 204.\end{aligned}$$

Density Function

Let $f(x)$ be a probability density function given by

$$f(x) = \begin{cases} cx^4(2-x), & 0 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of c?

Answer = number (3 significant figures)

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$\boxed{\text{Solution}}$

$$\begin{aligned}\int_{\mathbb{R}} f(x)\,dx &= \int_0^2 c x^4 (2-x)\,dx \\ &= c\left[2\int_0^2 x^4\,dx - \int_0^2 x^5\,dx\right] \\ &= c\left(2 \cdot \frac{1}{5} \cdot 2^5 - \frac{1}{6} \cdot 2^6\right) \\ &= \frac{64}{30} \cdot c = 1 \quad \therefore \quad c = \frac{30}{64}\end{aligned}$$

Probability Density?

Let $f(x)$ and $g(x)$ be two probability density functions. Then $1.7f(x) - 0.7g(x)$ is also a probability density function.

- (a) True
- ✗ (b) False

Let $f(x)$ and $g(x)$ be two probability density functions. Then $0.4f(x) + 0.6g(x)$ is also a probability density function.

- (a) False
- ✗ (b) True

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$\boxed{\text{Solution}}$ ① If $h(x) = \alpha f(x) + \beta g(x)$ with $\alpha+\beta=1$, $\alpha \geq 0$, $\beta \geq 0$ then $\int_{\mathbb{R}} h(x)\,dx = \alpha \cdot \int_{\mathbb{R}} f(x)\,dx + \beta \int_{\mathbb{R}} g(x)\,dx = \alpha+\beta = 1$ and $h(x) \geq 0$. Thus $h(x)$ is a PDF.

② If α or β is negative, $h(x)$ could be negative for some $x \in \mathbb{R}$. Thus h may not be a PDF in general.

Uniform Probability

Let U be a uniform random variable on $(6, 43)$. Find $\mathbb{P}(U \in (10, 12) \cup (35, 42))$.

Answer = number (3 significant figures)

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$\boxed{\text{Solution}}$ Since the PDF is $f(x) = \begin{cases} \frac{1}{43-6}, & 6 \leq x \leq 43 \\ 0, & \text{o.w} \end{cases}$

$$\mathbb{P}(U \in (10,12) \cup (35,42))$$

$$= \int_{10}^{12} \frac{1}{37}\,dx + \int_{35}^{42} \frac{1}{37}\,dx = \frac{1}{37} \cdot (2+7) = \frac{9}{37}$$