

Math 3215: Intro to Probability and Statistics

Exam 1, Summer 2023

Date: June 8, 2023

NAME: _____

ID: _____

READ THE FOLLOWING INFORMATION.

- This is a 75-minute.
- This exam contains 10 pages (including this cover page) and 7 questions. Total of points is 100.
- Books, notes, and other aids are not allowed.
- Show all steps to earn full credit.
- Do not unstaple pages. Loose pages will be ignored.

Question	Points	Score
1	15	
2	15	
3	10	
4	15	
5	15	
6	15	
7	15	
Total:	100	

1. Let A and B be two events such that $\mathbb{P}(A) = 1/2$, $\mathbb{P}(B) = 1/4$, and $\mathbb{P}(A \cup B) = 7/12$.

(a) (5 points) Find $\mathbb{P}(A \cap B)$.

(b) (5 points) Find $\mathbb{P}(A^c|B)$.

(c) (5 points) Is A and B independent? Justify your answer.

2. Consider an urn containing 12 balls, of which 5 are green and 7 are red. A sample of size 9 is to be drawn at random. Let A be the event that first 3 balls are green, and B the event that exactly 4 green balls are drawn.

(a) (8 points) If 9 balls are drawn **with replacement**, what is $\mathbb{P}(A|B)$?

(b) (7 points) If 9 balls are drawn **without replacement**, what is $\mathbb{P}(A|B)$? (Hint: You may use Bayes' formula.)

3. (10 points) A hat contains 10 coins, where 9 are fair but one is double-headed (always landing Heads). A coin is chosen uniformly at random. The chosen coin is flipped 4 times. Find the probability that the chosen coin is double-headed given that it lands Heads all 4 times.

4. Suppose X is a random variable taking values in $S = \{0, 1, 2, 3, \dots\}$ with PMF

$$f(k) = \begin{cases} c, & k = 0, \\ \frac{1}{3^k k!}, & k = 1, 2, \dots \end{cases}$$

(a) (5 points) Find the value of c that would make this a valid probability model.

(b) (5 points) Find $\mathbb{E}[X]$.

(c) (5 points) Find $\mathbb{E}[3^X]$.

5. Let X be a random variable with moment generating function given by $M(t) = c(1 - 2t)^{-5}$ for $t < \frac{1}{2}$ for some constant c .

(a) (5 points) Find the constant c .

(b) (5 points) Find the expectation of X .

(c) (5 points) Find the variance of X .

6. An urn contains 4 white and 5 black balls. We randomly choose 4 balls. If 2 of them are white and 2 are black, we stop. If not, we replace the balls in the urn and again randomly select 4 balls. This continues until exactly 2 of the 4 chosen are white.

(a) (7 points) What is the probability that we shall make exactly 10 selections?

(b) (8 points) What is the probability that we select 4 balls more than 5 times?

7. In a weekly lottery you have probability .05 of winning a prize with a single ticket. Suppose you buy 1 ticket per week for 20 weeks and let X be the number of winning tickets.

(a) (5 points) Find $E[X]$.

(b) (5 points) What is the probability that among the twenty tickets you buy more than 3 winning tickets? Use the corresponding tables to find an approximate value for this probability.

(c) (5 points) Using a Poisson approximation, write down an expression for the probability that among the twenty tickets you buy more than 3 winning tickets. Use the corresponding tables to find an approximate value for this probability.

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