Practice Problems for Exam 2

MATH 3215, Spring 2024

1. The PDF of Y is given by $f_Y(y) = \frac{c}{y^3}$ for $1 < y < \infty$ and otherwise 0.
   (a) Find the value of c so that f_Y is a PDF.
   (b) Compute $\mathbb{E}[Y]$.

2. Let X be a continuous random variable with the PDF
$$f_X(x) = \begin{cases} x+1, & -1 < x < 0, \\ 1-x, & 0 \leq x < 1. \end{cases}$$
   Find the CDF of X. Draw the graph of the CDF.

3. Let X be the total amount of medical claims (in \$ 100,000) of the employees of a company. Assume that the PDF $f(x) = 30x(1-x)^4$ for $0 < x < 1$.
   (a) Find $\mathbb{E}[X]$ and $\mathrm{Var}(X)$.
   (b) Find the probability that the total exceeds \$20,000.

4. Let X be the number of alpha particle emissions of carbon-14 that are counted by a Geiger counter each second. Assume that the distribution of X is Poisson with mean 5. Let W be the time in seconds before the third count is made.
   (a) What is the distribution of W?
   (b) Find $\mathbb{P}(W \leq 1)$.

5. A loss (in \$ 100,000) due to fire in a building has a PDF $f(x) = \frac{1}{6}e^{-x/6}$, $0 < x < \infty$. Find the conditional probability that the loss is greater than 8 given that it is greater than 5.

6. Let $X \sim N(650, 400)$.
   (a) Find $\mathbb{P}(600 \leq X < 660)$.
   (b) Find a constant $c > 0$ such that $\mathbb{P}(|X - 650| \leq c) = 0.95$.

7. Let $X \sim N(0, 4)$ and $W = X^2$. Find the PDF of W.

8. Let X be a random variable with the CDF
$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{x^2}{4}, & 0 \leq x < 1, \\ \frac{x+1}{4}, & 1 \leq x < 2, \\ 1, & x \geq 2. \end{cases}$$
   Find $\mathbb{E}[X], \mathrm{Var}(X), \mathbb{P}(\frac{1}{4} < X < 1), \mathbb{P}(X = 1), \mathbb{P}(X = \frac{1}{2}), \mathbb{P}(\frac{1}{2} \leq X < 2)$.

9. A loss X on a car has an exponential distribution with a mean of \$5000. If the loss X on a car is greater than the deductible of \$500, the difference $X - 500$ is paid to the owner of the car. Considering zero (if $X < 500$) as a possible payment, find the expectation of the payment.

10. Let X and Y be discrete random variables with joint PMF

$$f(x,y) = \frac{x+y}{21}, \qquad x = 1,2, \quad y = 1,2,3.$$

   (a) Find the marginal PMFs of X and Y.
   (b) Find $\mathbb{P}(X+Y \le 3)$.

11. Let X and Y be discrete random variables with joint PMF

$$f(0,0) = f(1,2) = 0.2, \quad f(0,1) = f(1,1) = 0.3.$$

   (a) Find $\mathrm{Cov}(X,Y)$.
   (b) Find the least square regression line.

12. Let X and Y be discrete random variables with joint PMF $f(x,y) = \frac{1}{9}$ for $(x,y) \in S$ where

$$S = \{(x,y) : 0 \le x \le 2, x \le y \le x+2, x, y \text{ are integers}\}.$$

   Find $f_X(x), f_{Y|X}(y|1), \mathbb{E}[Y|X=1]$, and $f_Y(y)$.

13. Suppose that X has a geometric distribution with parameter p. and suppose the conditional distribution of Y, given $X = x$, is Poisson with mean x. Find $\mathbb{E}[Y]$ and $\mathrm{Var}(Y)$.

14. Let X and Y have the joint PDF

$$f(x,y) = \frac{4}{3}, \qquad 0 < x < 1, \quad x^3 < y < 1$$

   and 0 otherwise. Find $\mathbb{P}(X > Y)$.

15. Let X be a uniform random variable over $(0,2)$ and Y given $X = x$ be $U(0, x^2)$.
   (a) Find the joint PDF $f(x,y)$ and marginal $f_Y(y)$.
   (b) Find $\mathbb{E}[Y|X]$ and $\mathbb{E}[X|Y]$.

16. A certain type of electrical motors is defective with probability 1/100. Pick 1000 motors and let X be the number of defective ones among these 1000 motors. Using a normal approximation, (with/without) mid-point correction, write down an expression for the probability that among the 1000 motors 13 or less are defective.