Quiz 4 Solution

MATH 461 §22

Normal Probability

Let $X \sim \mathrm{N}(-3, 8)$. Find $\mathbb{P}(-8.2 < X \leq 1.64)$.

You can use the following Normal Table.

Answer = number (3 significant figures)

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$\mu = -3$, $\sigma^2 = 8$ $\Rightarrow$ $X = \sqrt{8} \cdot Z - 3$, $Z \sim N(0,1)$

$$\begin{aligned}
\mathbb{P}(-8.2 < X \leq 1.64) &= \mathbb{P}(-8.2 < \sqrt{8}Z - 3 < 1.64) \\
&= \mathbb{P}\left(-\frac{5.2}{2\sqrt{2}} < Z < \frac{4.64}{2\sqrt{2}}\right) \\
&= \Phi\left(\frac{2.32}{\sqrt{2}}\right) - \Phi\left(-\frac{2.6}{\sqrt{2}}\right) \\
&\approx \Phi(1.640) + \Phi(1.838) - 1 \\
&\approx 0.9495 + 0.967 - 1 = \boxed{0.9165}
\end{aligned}$$

Exponential Distribution

Suppose that X is exponentially distributed with parameter 4. Consider the random variable $Y = e^{-2X}$.

a) (5 pt) Find $\mathbb{P}(Y \leqslant 0.98)$.

Answer = number (3 significant figures)

b) (5 pt) Let $f_Y(x)$ be the PDF of Y. Find $f_Y(0.98)$.

Answer = number (3 significant figures)

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For $0 \leq t \leq 1$,

$$\begin{aligned}
F_Y(t) &= \mathbb{P}(Y \leq t) = \mathbb{P}(e^{-2X} \leq t) = \mathbb{P}(-2X \leq \log t) \\
&= \mathbb{P}(X \geq -\tfrac{1}{2}\log t) = e^{-4\cdot(-\frac{1}{2}\log t)} = t^2.
\end{aligned}$$

Since $0 \leq e^{-2X} \leq 1$, $F_Y(t) = 0$ if $t > 1$ or $t < 0$.

$$f_Y(t) = \frac{d}{dt} F_Y(t) = \begin{cases} 2t & , \ 0 \le t \le 1 \\ 0 & , \ \text{o.w.} \end{cases}$$

$$\therefore \ P(Y \le 0.98) = F_Y(0.98) = (0.98)^2 = \boxed{0.9604}$$

$$f_Y(0.98) = 2 \cdot (0.98) = \boxed{1.96}$$

Joint PMF

Consider discrete random variables X and Y having joint pmf given by

$p(x,y)$	$y=3$	$y=5$	$y=7$	$y=9$
$x=2$	0.11	0.21	0.06	0.22
$x=4$	0.27	0.01	0.1	0.02

What is $\mathbb{P}(2X > Y)$?

Answer = number (2 significant figures)

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$$\mathbb{P}(2X > Y) = \mathbb{P}(X=2, Y=3) + \mathbb{P}(X=4, Y=3)$$
$$+ \mathbb{P}(X=4, Y=5) + \mathbb{P}(X=4, Y=7)$$
$$= 0.11 + 0.27 + 0.01 + 0.1 = \boxed{0.49}$$