# Practice for Final Exam

MATH 3215, Summer 2023

## 1 Exam Info

- Date: July 31, 2023
- Time: 8:00-10:50am (2hr 50min)
- There will be about 12 problems. No proof questions.
- No book, notes, or calculator are allowed. Tables for distributions (from appendix) will be provided.
- Coverage: Cumulative. The problems will be mostly from practice problems (for midterm 1,2, and the final) and midterm exams. There could be possibly minor modifications.

## 2 Problmes

1. Let $X_1, X_2, \cdots, X_7$ be an i.i.d. sequence of Poisson random variables with parameter $\lambda = 2$. Let $W = \sum_{i=1}^{7} X_i$. Find the mgf of $W$. How is $W$ distributed?
2. Suppose $X \sim N(1,4)$ and $Y \sim N(2,5)$ are independent normal random variables. Let $W = X + Y$. Find the mgf of $W$.
3. A certain type of electrical motors is defective with probability 1/100. Pick 1000 motors and let $X$ be the number of defective ones among these 1000 motors.
   (a) What is the probability that among the 1000 motors 13 or less are defective, i.e., what is $\mathbb{P}(X \leq 13)$.
   (b) Using a normal approximation, with mid-point correction, write down an expression for the probability that among the 1000 mortors 13 or less are defective. Use the corresponding tables (and $\sqrt{\frac{99}{10}} \approx 3.15$) to find an approximate value for thsi probability
4. A fair die will be rolled 720 times independently.
   (a) What is the probability that among the 720 rolls the number 6 will appear between 135 and 150 times inclusively? That is, what is $\mathbb{P}(135 \leq X \leq 150)$? Write down the probability without using the tables and approximations.
   (b) Using a normal approximation, **without mid-point correction**, write down an expression for the probability that among the 720 rolls the number 6 will appear between 135 and 150 times inclusively. Use the corresponding tables to find an approximate value for this probability.

(c) Using a normal approximation, **with mid-point correction**, write down an expression for the probability that among the 720 rolls the number 6 will appear between 135 and 150 times inclusively. Use the corresponding tables to find an approximate value for this probability.

5. If $X$ is a random variable with mean 3 and variance 16, use Chebyshev's inequality to find

   (a) A lower bound for $\mathbb{P}(23 < X < 43)$.

   (b) An upper bound for $\mathbb{P}(|X-31| \geq 14)$.

6. Let $\overline{X}$ be the mean of a random sample of size $n = 15$ from a distribution with mean $\mu = 80$ and variance $\sigma^2 = 60$. Use Chebvshev's inequality to find a lower bound for $\mathbb{P}(75 < X < 85)$.

7. Let $W_1 < W_2 < \cdots < W_{10}$ be the order statistics of $n$ independent observations from a $U(0,1)$ distribution.

   (a) Find the pdfs of $W_1$ and $W_{10}$.

   (b) Find $\mathbb{E}(W_1)$ and $\mathbb{E}[W_{10}]$.

8. Let $Y_1 < Y_2 < \cdots < Y_5$ be the order statistics of a random sample of size 5 from a distribution with pdf $f(x) = e^{-x}$, $0 < x < \infty$.

   (a) Find the pdf of $Y_3$.

   (b) Find the pdf of $U = e^{-Y_3}$.

9. Suppose that $X$ is a discrete random variable with pmf

$$f(x) = \frac{2+\theta(2-x)}{6}, \qquad x = 1,2,3,$$

   where the unknown parameter $\theta$ belongs to the parameter space $\Omega = \{-1,0,1\}$. Suppose further that a random sample $X_1, X_2, X_3, X_4$ is taken from this distribution, and the four observed values are $(x_1,x_2,x_3,x_4) = (3,2,3,1)$. Find the maximum likelihood estimate of $\theta$.

10. A random sample of size 16 from the normal distribution $N(\mu, 25)$ yielded $\overline{X} = 73.8$. Find a 95% confidence interval for $\mu$.

11. To determine the effect of 100% nitrate on the growth of pea plants, several specimens were planted and then watered with 100% nitrate every day. At the end of two weeks, the plants were measured. Here are data on seven of them:

$$17.5, 14.5, 15.2, 14.0, 17.3, 18.0, 13.8$$

   Assume that these data are a random sample from a normal distribution $N(\mu, \sigma^2)$.

   (a) Find the value of a point estimate of $\mu$.

   (b) Find the value of a point estimate of $\sigma$.

   (c) Give the endpoints for a 90% confidence interval for $\mu$.