MATH 403 FALL 2021: QUIZ 1 SOLUTION
DATE: SEP 1, 2021

(a) (4 points) Let $P, A, B \in \mathbb{R}^2$. Write down the definition of $P \in \ell_{AB}$.

Solution. $P \in \ell_{AB}$ if and only if there exists $t \in \mathbb{R}$ such that $P = (1-t)A + tB$.

(b) (4 points) Let $A, B, C, D \in \mathbb{R}^2$. Write down the definition of $\ell_{AB} \mathbin{/\!/} \ell_{CD}$.

Solution. $\ell_{AB} \mathbin{/\!/} \ell_{CD}$ if and only if there exists t such that $(D-C) = t(B-A)$.

(c) (2 points) Let $A, B, C, D \in \mathbb{R}^2$. Assume ℓ_{AB} and ℓ_{CD} are parallel. Show that if there exists a point $P \in \mathbb{R}^2$ such that $P \in \ell_{AB} \cap \ell_{CD}$, then $C \in \ell_{AB}$.

Solution. Since ℓ_{AB} and ℓ_{CD} are parallel, there exists $t \in \mathbb{R}$ such that $(D-C) = t(B-A)$. Since $P \in \ell_{AB} \cap \ell_{CD}$, we have $r, s \in \mathbb{R}$ such that $P = (1-r)A + rB = (1-s)C + sD$. Thus,

$$A + r(B-A) = C + s(D-C) = C + ts(B-A),$$
$$C = A + (r-ts)(B-A) = (1-(r-ts))A + (r-ts)B.$$

By definition, we conclude that $C \in \ell_{AB}$.