

## MATH 403 FALL 2021: HOMEWORK 9

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1. Exercise 3.14 (Hint: Use the orthogonal projection of  $Y$  to  $X$ .)
2. Exercise 3.16
3. Exercise 3.17
4. Exercise 3.18
5. For  $X = (x_1, x_2)$  and  $Y = (y_1, y_2)$ , we define

$$d_\infty(X, Y) := \max\{|x_1 - y_1|, |x_2 - y_2|\}.$$

Show that  $d_\infty(\cdot, \cdot)$  satisfies the following:

- (a)  $d_\infty(X, Y) = d_\infty(Y, X)$  for any  $X, Y \in \mathbb{R}^2$ .
  - (b)  $d_\infty(X, Y) \geq 0$  for any  $X, Y \in \mathbb{R}^2$  and  $d_\infty(X, Y)$  equals to zero if and only if  $X = Y$ .
  - (c)  $d_\infty(X, Z) \leq d_\infty(X, Y) + d_\infty(Y, Z)$  for any  $X, Y, Z \in \mathbb{R}^2$ .
6. (Continued) Draw the set  $\{X : d_\infty(X, D) = 2\}$  for  $D = (2, 3)$  in the plane.