MATH 403 FALL 2021: HOMEWORK 10

1. Let α be an isometry and ℓ be a line. Show that $\sigma_{\alpha(\ell)} = \alpha\sigma_\ell\alpha^{-1}$.

2. Exercise 4.2

3. Exercise 4.5: Let α be an isometry. Suppose that $\alpha\sigma_\ell = \sigma_\ell\alpha$ holds for any line ℓ. Show that α is the identity map.

4. Exercise 4.7: Let ℓ_1, ℓ_2 be lines with $\ell_1 \perp \ell_2$. Show that $\sigma_{\ell_1}\sigma_{\ell_2} = \sigma_C$ for some $C \in \mathbb{R}^2$.

5. Exercise 4.9

6. Exercise 4.10

7. Exercise 4.11

8. Exercise 4.12

9. Exercise 4.13

10. Let α be a rotation with center $C \in \mathbb{R}^2$ through the oriented angle $\theta \in (-\pi, \pi]$ and ℓ be a line. Suppose α is not the identity map and $\alpha(\ell) = \ell$. Show that α is the central reflection in C.