**MATH 403 FALL 2021: QUIZ 8 SOLUTION**
**DATE: NOV 3, 2021**

(a) (5 points) Write down the statement of Thales Theorem.

**Solution.** Consider a triangle $\triangle ABC$. Let $C'$ be the midpoint of $A$ and $B$. Let $\mathcal{S}$ be the circle with center $C'$ and radius $\frac{1}{2}|\overline{AB}|$. (That is, the line segment $\overline{AB}$ is the diameter of $\mathcal{S}$.) Then, $C \in \mathcal{S}$ if and only if $\overline{AC}$ is perpendicular to $\overline{BC}$.

(b) (5 points) Let $ABCD$ be a rectangle. Show that there exists a circle containing $ABCD$.

**Solution.** Consider a circle $\mathcal{S}$ whose diameter is $\overline{AC}$. Since $\ell_{AB} \perp \ell_{BC}$, Thales theorem yields $B \in \mathcal{S}$. Similarly, $\ell_{AD} \perp \ell_{DC}$ implies $D \in \mathcal{S}$.