MATH 403 FALL 2021: HOMEWORK 2 SOLUTION

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DUE DATE: SEP 10, 2021

1. Exercise 1.14

Solution: (a): Let $n \in \mathbb{N}$ and $A_i \in \mathbb{R}^2$ for $i = 1, 2, \cdots, n$. Let G_i be the centroid of $A_1, A_2, \cdots, A_n$ except A_i, and G the centroid of A_i, $i = 1, 2, \cdots, n$. Then,

$$G \in \bigcap_{i=1}^{n} \ell_{A_i G_i}.$$

(b): Let $S = \sum_{i=1}^{n} A_i$, then $G_i = \frac{1}{n-1}(S - A_i)$. Since

$$G = \frac{S}{n} = \frac{1}{n}(S - A_i) + \frac{A_i}{n} = \frac{n-1}{n}\left(\frac{1}{n-1}(S - A_i)\right) + \frac{A_i}{n} = \frac{n-1}{n}G_i + \frac{1}{n}A_i,$$

we have $G \in \ell_{A_i G_i}$ for all $i = 1, 2, \cdots, n$.

2. Exercise 1.15 (a)

Solution: Let M_1 be the midpoint of A, B, M_2 of C, D, N_1 of A, C, N_2 of B, D, L_1 of A, D, L_2 of B, C. That is,

$$M_1 = \frac{A+B}{2}, \quad M_2 = \frac{C+D}{2},$$
$$N_1 = \frac{A+C}{2}, \quad N_2 = \frac{B+D}{2},$$
$$L_1 = \frac{A+D}{2}, \quad L_2 = \frac{B+C}{2}.$$

Suppose $P \in \ell_{M_1 M_2} \cap \ell_{N_1 N_2}$, then there exist $t, s \in \mathbb{R}$ such that

$$P = \frac{1-t}{2}(A+B) + \frac{t}{2}(C+D) = \frac{1-s}{2}(A+C) + \frac{s}{2}(B+D)$$

One can see that these equations are satisfied when $t = s = \frac{1}{2}$. Furthermore, let $G = \frac{1}{4}(A+B+C+D)$ then

$$G = \frac{1}{2}L_1 + \frac{1}{2}L_2$$

so that $G \in \ell_{M_1 M_2} \cap \ell_{N_1 N_2} \cap \ell_{L_1 L_2}$.

3. Exercise 1.15 (b)

Solution: Checking the identity is straightforward. The geometric meaning is that (1) the diagonal $\overline{AC}$ is parallel to $\overline{L_2 M_1}$ and $\overline{M_2 L_1}$, and (2) the length of the diagonal $\overline{AC}$ is twice the length of $\overline{L_2 M_1}$ and $\overline{M_2 L_1}$.

4. Exercise 1.16 (a)

Solution:

$$E + G = \frac{1}{3}(A + 2B + C + 2D) = F + H.$$

5. Exercise 1.16 (b)

Solution: We have

$$E + G = \frac{1}{a+b}(aA + bB) + \frac{1}{c+d}(cC + dD),$$
$$F + H = \frac{1}{b+c}(bB + cC) + \frac{1}{d+a}(dD + aA).$$

If $a = c$ and $b = d$, then $E + G = F + H$.