# MATH 403 FALL 2021: HOMEWORK 8

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1. Consider $\triangle ABC$. Let $A', B', C'$ be the midpoints of the sides of $\triangle ABC$. Let $H$ be the orthocenter. Let $A'' = \frac{1}{2}(H + A)$, $B'' = \frac{1}{2}(H + B)$, and $C'' = \frac{1}{2}(H + C)$. Let $\mathcal{S}$ a circle with the line segment $\overline{A'A''}$ as diameter. Show that $B', B'', C', C'' \in \mathcal{S}$ using Exercise 3.7 and Thales' Theorem (Theorem 3.6).
2. (Continued) Show that line segments $\overline{B'B''}$ and $\overline{C'C''}$ are also diameters of $\mathcal{S}$. Let $D, E, F$ be the feet of the altitudes of $\ell_A, \ell_B, \ell_C$. Using Thales' Theorem, deduce that $D, E, F \in \mathcal{S}$.
3. Let $A, B \in \mathbb{R}^2$ with $A \neq B$. Find the minimum of $d(P, A) + d(P, B)$ for $P \in \mathbb{R}^2$.
4. Exercise 3.12
5. Let $X, Y, A \in \mathbb{R}^2 \setminus \{O\}$. Suppose $X \cdot A = 0$ and $Y \cdot A = 0$. Show that $O, X, Y$ are collinear.