MATH 403 FALL 2021: EXAM 3 PRACTICE PROBLEMS

1. Definitions

(a) Rhombus, Rectangle, Circle

(b) Perpendicular bisectors, Altitude, The foot of altitude, Circumcenter, Orthocenter

(c) Distance between X and Y, The length of X

(d) Orthogonal projection, Angle, Determinant

(e) An isometry, A linear isometry, A linear map

2. Statements of Theorems (and proofs)

(a) Pythagoras Theorem, Thales Theorem, Parallelogram Law

(b) Triangle inequality, Cauchy–Schwarz inequality

(c) Nine point circle theorem

3. Examples

Give an example, or explain why no such example exists.

(a) $X, Y \neq O$, and $|X + Y| = |X - Y|$.

(b) $|X| = 4$, $|Y| = 2$, and $X \cdot Y = 9$.

(c) A triangle $\triangle ABC$ whose orthocenter and circumcenter coincide.

(d) $|X| = 1$, $|Y| = 2$, and $|X - Y| = 4$.

4. Proof or Disproof

4.1. **Scalar Product.**

(a) If $X \cdot Y = 0$ and $Y \neq O$, then $X = O$.

(b) If $|X| = 3$, $|Y| = 4$, then $|X \cdot Y| \leqslant 12$.

(c) If $\alpha : \mathbb{R}^2 \to \mathbb{R}^2$ satisfies $X \cdot Y = \alpha(X) \cdot \alpha(Y)$ for all X, Y, then $|\alpha(X) - \alpha(Y)| = |X - Y|$ for all X, Y.

(d) If X is perpendicular to Y, then $|X + Y|^2 = |X|^2 + |Y|^2$.

(e) For $X \neq O$, the projection $\mathrm{Proj}_Y X$ is perpendicular to X.

4.2. **Triangles.**

(a) The altitudes (the Perpendicular bisectors) of a triangle are concurrent.

(b) Formulas for the area of a triangle.

(c) If X is on the perpendicular bisector of $\overline{AB}$, then $|X - A| = |X - B|$.

(d) In the setting of Nine point circle theorem, $|A' - A''| = |B' - B''| = |C' - C''|$.

4.3. **Isometry.**

(a) The set of all isometries forms a group.

(b) If α is an isometry, then $(\alpha(X) - \alpha(Z)) \cdot (\alpha(Y) - \alpha(Z)) = (X - Z) \cdot (Y - Z)$ for all X, Y, Z.

(c) Every dilatation is an isometry.