MATH 403 FALL 2021: QUIZ 9 SOLUTION
DATE: NOV 10, 2021

(a) (4 points) Write down the definition of an isometry.

Solution. An isometry is a distance preserving map. That is, a map $\alpha : \mathbb{R}^2 \to \mathbb{R}^2$ is an isometry if and only if

$$d(\alpha(X), \alpha(Y)) = d(X, Y) \quad \text{for all } X, Y.$$

(b) (3 points) Let $\alpha : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $\alpha((x, y)) = (y + 1, -x)$. Is the map α an isometry? Justify your answer.

Solution. Yes, it is an isometry. For $X = (x_1, x_2)$ and $Y = (y_1, y_2)$,

$$|\alpha(X) - \alpha(Y)| = |(x_2 + 1, -x_1) - (y_2 + 1, -y_1)| = \sqrt{(x_2 - y_2)^2 + (y_1 - x_1)^2} = |X - Y|.$$

(c) (3 points) Is the map α linear? Justify your answer.

Solution. No, it is not linear since $\alpha(O) \neq O$.