MATH 403 FALL 2021: QUIZ 3 SOLUTION
DATE: SEP 15, 2021

Let $(a, A), (b, B), (c, C)$ be mass-points with $a, b, c > 0$.

(a) (4 points) What is the centroid G of $(a, A), (b, B), (c, C)$?

Solution. $G = \dfrac{aA + bB + cC}{a + b + c}$.

(b) (4 points) Let $A' = \dfrac{bB + cC}{b + c}$. Show that $G \in \ell_{AA'}$.

Solution. We have

$$\begin{aligned} G &= \frac{aA + bB + cC}{a + b + c} \\ &= \frac{a}{a + b + c}A + \frac{b + c}{a + b + c}\left(\frac{bB + cC}{b + c}\right) \\ &= \frac{a}{a + b + c}A + \frac{b + c}{a + b + c}A'. \end{aligned}$$

Since $a/(a + b + c) + (b + c)/(a + b + c) = 1$, we have $G \in \ell_{AA'}$.

(c) (2 points) What is the meaning of $\dfrac{G - A'}{A - A'} = t$? Find t.

Solution. It means that $(G - A') = t(A - A')$. One can see that

$$\begin{aligned} G - A' &= \frac{a}{a + b + c}A + \frac{b + c}{a + b + c}A' - A' \\ &= \frac{a}{a + b + c}(A - A'). \end{aligned}$$

Thus $t = \dfrac{a}{a + b + c}$.