

MATH 403 FALL 2021: QUIZ 5 SOLUTION

DATE: OCT 6, 2021

(a) (4 points) Give the definition of a group.

Solution. Let G be a set equipped with an operation $(x, y) \mapsto xy \in \mathcal{G}$ for all $x, y \in G$. We say G is a group with the operation if

- (a) There exists an element e in G such that $ex = xe = x$ for all $x \in G$. We call e the identity.
- (b) For every $x \in G$, there is an element $x^{-1} \in G$ such that $xx^{-1} = x^{-1}x = e$.
- (c) For all $x, y, z \in G$, we have $x(yz) = (xy)z$.

(b) (3 points) Is the set of real numbers with addition a group? Justify your answer.

Solution. Yes. For any real numbers x, y , $x + y$ is also a real number. So the operation is well-defined.

- (a) The identity is 0, which is a real number.
- (b) For every $x \in \mathbb{R}$, $-x$ is the inverse, since $x + (-x) = 0$.
- (c) For all $x, y, z \in \mathbb{R}$, we have $x + (y + z) = (x + y) + z$.

(c) (3 points) Is the set of real numbers with multiplication a group? Justify your answer.

Solution. No. Since $x \cdot 0 = 0$ for all $x \in \mathbb{R}$, there is no identity.