MATH 403 FALL 2021: QUIZ 6 SOLUTION
DATE: OCT 13, 2021

Let $C, D \in \mathbb{R}^2$ with $C \neq D$.

(a) (4 points) For $X \in \mathbb{R}^2$, compute $\delta_{D,\frac{1}{2}} \circ \delta_{C,\frac{1}{2}}(X)$.

Solution.

$$\delta_{D,\frac{1}{2}} \circ \delta_{C,\frac{1}{2}}(X) = \delta_{D,\frac{1}{2}}(\frac{1}{2}C + \frac{1}{2}X)$$
$$= \frac{1}{2}D + \frac{1}{4}C + \frac{1}{4}X.$$

(b) (3 points) Find $P \in \mathbb{R}^2$ and $r \in \mathbb{R}$ such that $\delta_{D,\frac{1}{2}} \circ \delta_{C,\frac{1}{2}} = \delta_{P,r}$.

Solution.

$$\delta_{D,\frac{1}{2}} \circ \delta_{C,\frac{1}{2}}(X) = \frac{1}{2}D + \frac{1}{4}C + \frac{1}{4}X$$
$$= \frac{3}{4}\left(\frac{2}{3}D + \frac{1}{3}C\right) + \frac{1}{4}X$$
$$= \delta_{P,\frac{1}{4}}(X)$$

where $P = \frac{2}{3}D + \frac{1}{3}C$.

(c) (3 points) Find all fixed points of the map $\delta_{D,\frac{1}{2}} \circ \delta_{C,\frac{1}{2}}$.

Solution. The map $\delta_{P,\frac{1}{4}}$ has the unique fixed point P.