Chapter 1. Overview and Descriptive Statistics

Math 3670 Summer 2024

Georgia Institute of Technology

Section 1.
Populations, Samples, and Processes

Population and Sample

Population: A well-defined collection of objects.
Ex: All individuals who received a B.S. in engineering during the most recent academic year

Sample: A subset of the population selected in a prescribed way.
Ex: Select a sample of last year's engineering graduates to obtain feedback about the quality of the engineering curricula

Variable: Any characteristic whose value may change from one object to another in the population
Ex: Brand of calculator owned by a student

Univariate, Bivariate, Multivariate Data

Descriptive Statistics

To summarize and describe important features of the data

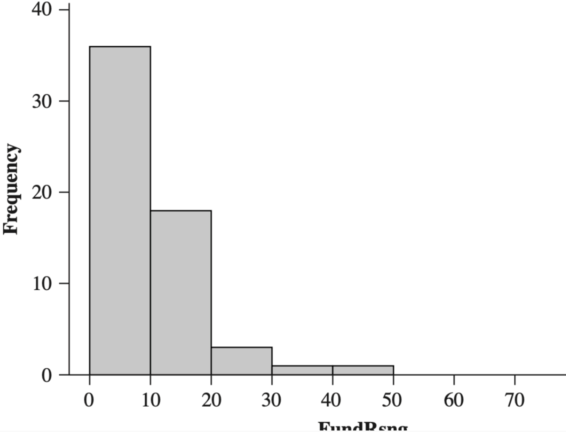
1. Graphical methods: Boxplots, Histograms, Scatter plots, etc.
2. Numerical methods: means, standard deviations, correlation coefficients, etc.

Example

Charity is a big business in the United States. The Web site charitynavigator.com gives information on roughly 5500 charitable organizations, and there are many smaller charities that fly below the navigator's radar screen. Some charities operate very efficiently, with fundraising and administrative expenses that are only a small percentage of total expenses, whereas others spend a high percentage of what they take in on such activities. Here is data on fundraising expenses as a percentage of total expenditures for a random sample of 60 charities:

6.1	12.6	34.7	1.6	18.8	2.2	3.0	2.2	5.6	3.8
2.2	3.1	1.3	1.1	14.1	4.0	21.0	6.1	1.3	20.4
7.5	3.9	10.1	8.1	19.5	5.2	12.0	15.8	10.4	5.2
6.4	10.8	83.1	3.6	6.2	6.3	16.3	12.7	1.3	0.8
8.8	5.1	3.7	26.3	6.0	48.0	8.2	11.7	7.2	3.9
15.3	16.6	8.8	12.0	4.7	14.7	6.4	17.0	2.5	16.2

```
Stem-and-leaf of FundRsng N = 60
Leaf Unit = 1.0
    0 | 011111222233333344
    0 | 5555666666778888
    1 | 0001222244
    1 | 55666789
    2 | 01
    2 | 6
    3 | 4
    3 |
    4 |
    4 | 8
    5 |
    5 |
    6 |
    6 |
    7 |
    7 |
    8 | 3
```



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Overview

Section 2.
Pictorial and Tabular Methods in Descriptive Statistics

Consider a numerical data set $x_1, x_2, \cdots, x_n$ for which each x_i consists of at least two digits.

How to construct Stem-and-Leaf Displays

1. Select one or more leading digits for the stem values. The trailing digits become the leaves.
2. List possible stem values in a vertical column.
3. Record the leaf for each observation beside the corresponding stem value.
4. Indicate the units for stems and leaves someplace in the display.

Example

0	4
1	1345678889
2	1223456666777889999
3	0112233344555666677777888899999
4	111222222334444556666666777888888999
5	00111222223345566666777788889 9
6	01111244455666778

Stem: tens digit
Leaf: ones digit

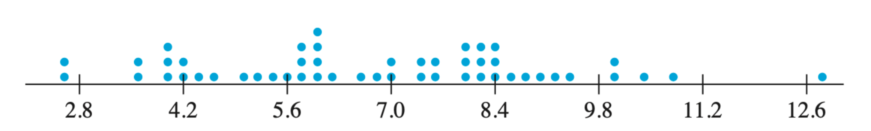
Figure 1.4 Stem-and-leaf display for the percentage of binge drinkers at each of the 140 colleges

Information from Stem-and-Leaf Displays

1. identification of a typical or representative value
2. extent of spread about the typical value
3. presence of any gaps in the data
4. extent of symmetry in the distribution of values
5. number and location of peaks
6. presence of any outlying values

Dotplots

10.8	6.9	8.0	8.8	7.3	3.6	4.1	6.0	4.4	8.3
8.1	8.0	5.9	5.9	7.6	8.9	8.5	8.1	4.2	5.7
4.0	6.7	5.8	9.9	5.6	5.8	9.3	6.2	2.5	4.5
12.8	3.5	10.0	9.1	5.0	8.1	5.3	3.9	4.0	8.0
7.4	7.5	8.4	8.3	2.6	5.1	6.0	7.0	6.5	10.3



Definitions

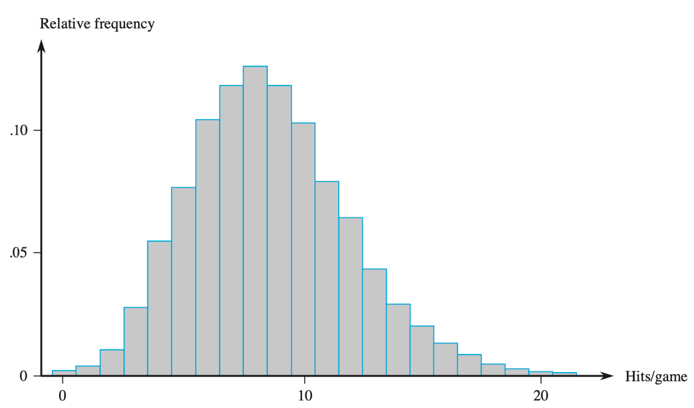
A numerical variable is **discrete** if its set of possible values either is finite or else can be listed in an infinite sequence (one in which there is a first number, a second number, and so on).

A numerical variable is **continuous** if its possible values consist of an entire interval on the number line.

Example (Discrete Data)

Hits/Game	Number of Games	Relative Frequency	Hits/Game	Number of Games	Relative Frequency
0	20	.0010	14	569	.0294
1	72	.0037	15	393	.0203
2	209	.0108	16	253	.0131
3	527	.0272	17	171	.0088
4	1048	.0541	18	97	.0050
5	1457	.0752	19	53	.0027
6	1988	.1026	20	31	.0016
7	2256	.1164	21	19	.0010
8	2403	.1240	22	13	.0007
9	2256	.1164	23	5	.0003
10	1967	.1015	24	1	.0001
11	1509	.0779	25	0	.0000
12	1230	.0635	26	1	.0001
13	834	.0430	27	1	.0001
				19,383	1.0005

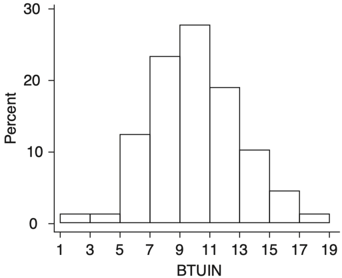
Example (Discrete Data)



Histograms

Example (Continuous Data)

Class	1−<3	3−<5	5−<7	7−<9	9−<11	11−<13	13−<15	15−<17	17−<19
Frequency	1	1	11	21	25	17	9	4	1
Relative frequency	.011	.011	.122	.233	.278	.189	.100	.044	.011

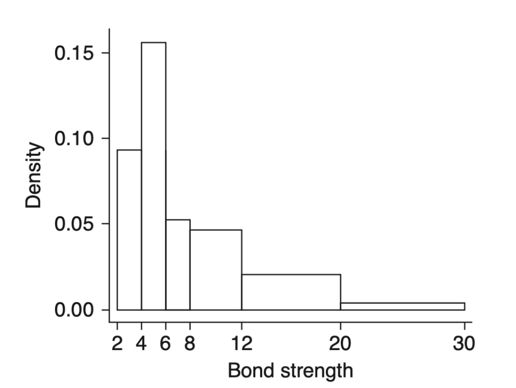


Example (Continuous Data)

11.5	12.1	9.9	9.3	7.8	6.2	6.6	7.0	13.4	17.1	9.3	5.6
5.7	5.4	5.2	5.1	4.9	10.7	15.2	8.5	4.2	4.0	3.9	3.8
3.6	3.4	20.6	25.5	13.8	12.6	13.1	8.9	8.2	10.7	14.2	7.6
5.2	5.5	5.1	5.0	5.2	4.8	4.1	3.8	3.7	3.6	3.6	3.6

Class	2–<4	4–<6	6–<8	8–<12	12–<20	20–<30
Frequency	9	15	5	9	8	2
Relative frequency	.1875	.3125	.1042	.1875	.1667	.0417
Density	.094	.156	.052	.047	.021	.004

Example (Continuous Data)



Section 3.
Measures of Locations

Definition

Consider a data set $x_1, x_2, \cdots, x_n$.

The sample mean is defined by

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

Example

Example

People not familiar with classical music might tend to believe that a composer's instructions for playing a particular piece are so specific that the duration would not depend at all on the performer(s).

However, there is typically plenty of room for interpretation, and orchestral conductors and musicians take full advantage of this.

Selected a sample of 12 recordings of Beethoven's Symphony #9 yielding the following durations (min) listed in increasing order:

$$62.3, 62.8, 63.6, 65.2, 65.7, 66.4, 67.4, 68.4, 68.8, 70.8, 75.7, 79.0$$

The sample mean is $\overline{x} = 816.1/12 = 68.01$.

The Median

Definition

The sample median is obtained by first ordering the n observations from smallest to largest.

If the number of the observation is even,

If the number of the observation is odd,

Example

People not familiar with classical music might tend to believe that a composer's instructions for playing a particular piece are so specific that the duration would not depend at all on the performer(s).

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$$62.3, 62.8, 63.6, 65.2, 65.7, 66.4, 67.4, 68.4, 68.8, 70.8, 75.7, 79.0$$

The sample median $\widetilde{x}$ is

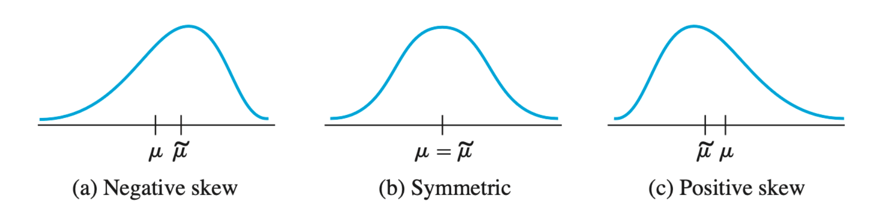
Population Mean and Median

Definition

The population mean is

$$\mu =$$

The population median $\tilde{\mu}$ is the median from the whole population.



Quartiles and Percentiles

Definition

Quartiles: divide the number of data points into four equal parts.

Percentiles: divide the number of data points into 100 equal parts.

Example

$$62.3, 62.8, 63.6, 65.2, 65.7, 66.4, 67.4, 68.4, 68.8, 70.8, 75.7, 79.0$$

Trimmed Means

Definition

A trimmed mean a trimming percentage of $\alpha\%$ is the mean of the data set after removing the smallest $\alpha\%$ and the largest $\alpha\%$.

Example

$$62.3, 62.8, 63.6, 65.2, 65.7, 66.4, 67.4, 68.4, 68.8, 70.8, 75.7, 79.0$$

Exercise

The May 1, 2009 issue of The Montclarian reported the fol- lowing home sale amounts for a sample of homes in Alameda, CA that were sold the previous month (1000s of \$):

$$590, 815, 575, 608, 350, 1285, 408, 540, 555, 679.$$

The sum is 6405.

1. Calculate and interpret the sample mean and median.
2. Suppose the 6th observation had been 985 rather than 1285. How would the mean and median change?
3. Calculate a 10% trimmed mean.

Section 4.
Measures of Variability

Example

Consider the two data sets

$$\begin{aligned} \text{Data 1:} \quad & 10, 20, 30, 40, 50, 60, 70, \\ \text{Data 2:} \quad & 30, 35, 37, 40, 43, 45, 50. \end{aligned}$$

The Sample Variance

Definition

The sample variance is defined by

$$s^2 = \frac{S_{xx}}{n-1} =$$

The sample standard deviation is

$$s = \sqrt{s^2}.$$

The Sample Variance

Suppose the population consists of $x_1, x_2, \cdots, x_N$.

Definition

The population variance is defined by

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2.$$

Example

Example

Car	x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	
1	27.3	−5.96	35.522	
2	27.9	−5.36	28.730	
3	32.9	−0.36	0.130	
4	35.2	1.94	3.764	
5	44.9	11.64	135.490	
6	39.9	6.64	44.090	
7	30.0	−3.26	10.628	
8	29.7	−3.56	12.674	
9	28.5	−4.76	22.658	
10	32.0	−1.26	1.588	
11	37.6	4.34	18.836	
	$\sum x_i = 365.9$	$\sum(x_i - \bar{x}) = .04$	$\sum(x_i - \bar{x})^2 = 314.106$	$\bar{x} = 33.26$

Computing Formula for s^2

Proposition

$$S_{xx} = \sum_{i=1}^{n} (x_i - \overline{x})^2 =$$

Example

Example

Consider the following data:

2.1389	2.8132	2.4451	2.4660	2.6038	2.4186
3.8592	2.1988	2.3529	2.2028	2.7468	1.5104
2.1987	2.5252	2.8462	2.2722	2.2026	2.0153

Knowing

$$\sum_{i=1}^{18} x_i = 43.8166, \qquad \sum_{i=1}^{18} x_i^2 = 110.5081,$$

find the sample mean and variance.

Boxplots

Order the n observations from smallest to largest and separate the smallest half from the largest half.

The median $\widetilde{x}$ is included in both halves if n is odd.

Then the lower fourth is the median of the smallest half and the upper fourth is the median of the largest half.

A measure of spread that is resistant to outliers is the **fourth spread** f_s, given by

$$f_s =$$

Example

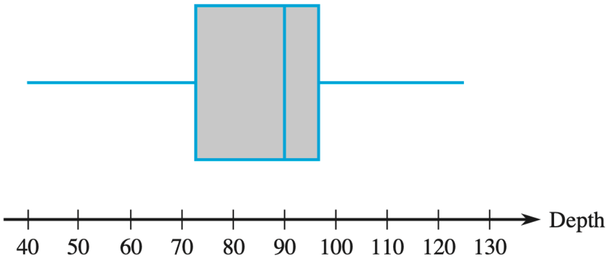
Consider the data

40	52	55	60	70	75	85	85	90	
90	92	94	94	95	98	100	115	125	125

Then,

1. The smallest x_i:
2. The lower fourth:
3. The median:
4. The upper fourth:
5. The largest x_i:

Example



Variable	N	Mean	Median	TrMean	StDev	SE Mean
depth	19	86.32	90.00	86.76	23.32	5.35

Variable	Minimum	Maximum	Q1	Q3
depth	40.00	125.00	70.00	98.00

Outliers

Definition

Any observation farther than $1.5f_s$ from the closest fourth is an **outlier**.

An outlier is **extreme** if it is more than $3f_s$ from the nearest fourth, and it is **mild** otherwise.

Example

The Clean Water Act and subsequent amendments require that all waters in the United States meet specific pollution reduction goals to ensure that water is "fishable and swimmable." The article "Spurious Correlation in the USEPA Rating Curve Method for Estimating Pollutant Loads" (*J. of Environ. Engr.,* 2008: 610–618) investigated various techniques for estimating pollutant loads in watersheds; the authors "discuss the imperative need to use sound statistical methods" for this purpose. Among the data considered is the following sample of TN (total nitrogen) loads (kg N/day) from a particular Chesapeake Bay location, displayed here in increasing order.

9.69	13.16	17.09	18.12	23.70	24.07	24.29	26.43
30.75	31.54	35.07	36.99	40.32	42.51	45.64	48.22
49.98	50.06	55.02	57.00	58.41	61.31	64.25	65.24
66.14	67.68	81.40	90.80	92.17	92.42	100.82	101.94
103.61	106.28	106.80	108.69	114.61	120.86	124.54	143.27
143.75	149.64	167.79	182.50	192.55	193.53	271.57	292.61
312.45	352.09	371.47	444.68	460.86	563.92	690.11	826.54
1529.35							

$\tilde{x} = 92.17$ lower $4^{th} = 45.64$ upper $4^{th} = 167.79$

$f_s = 122.15$ $1.5f_s = 183.225$ $3f_s = 366.45$