MATH 403 FALL 2021: QUIZ 7 SOLUTION
DATE: OCT 27, 2021

Let $A, B, C, D \in \mathbb{R}^2$ with $A \neq B$ and $C \neq D$.

(a) (5 points) Write the definition that two lines ℓ_{AB} and ℓ_{CD} are perpendicular.

Solution. Two lines ℓ_{AB} and ℓ_{CD} are perpendicular if $(A - B) \cdot (C - D) = 0$.

(b) (5 points) Let α be a dilatation (that is, either a translation or a central dilatation). Suppose ℓ_{AB} and ℓ_{CD} are perpendicular. Show that $\alpha(\ell_{AB})$ is also perpendicular to ℓ_{CD}.

Solution. We already know that $\alpha(\ell_{AB}) = \ell_{\alpha(A)\alpha(B)}$ and $\alpha(A) - \alpha(B) = t(A - B)$ for some t. Since the two lines are perpendicular, we have

$$(\alpha(A) - \alpha(B)) \cdot (C - D) = t(A - B) \cdot (C - D) = 0,$$

which completes the proof.